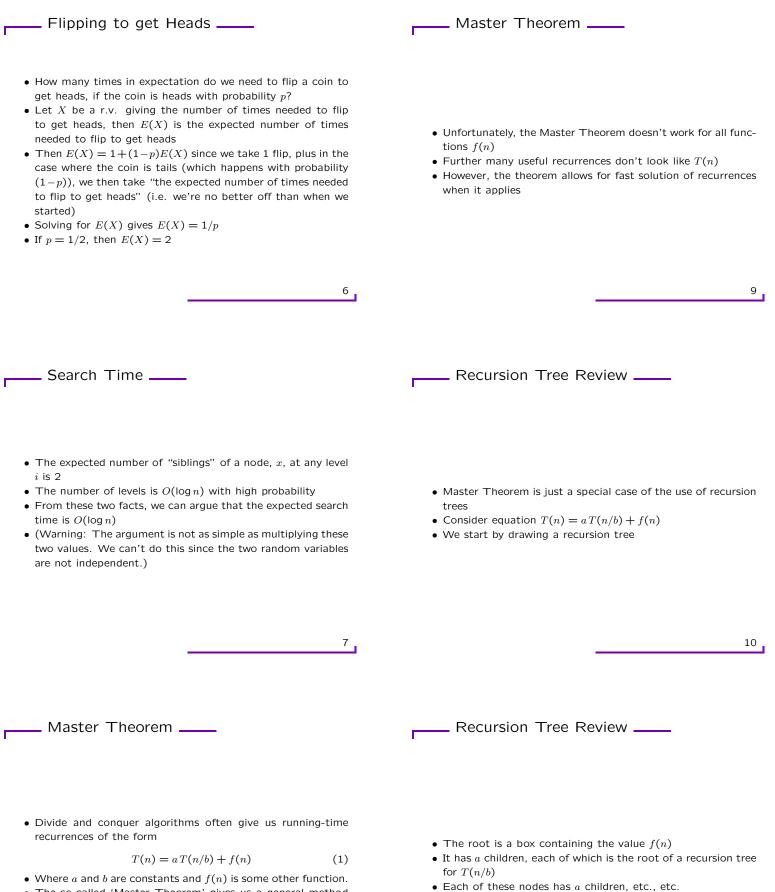
\_\_\_\_\_ Height of Skip List \_\_\_\_\_ • Q: What is the probability that any of the nodes exceed height  $k \log n$ ? CS 361, Lecture 27 • A: We want  $P(X_1 \ge k \log n \text{ or } X_2 \ge k \log n \text{ or } \dots \text{ or } X_n \ge k \log n)$ Jared Saia University of New Mexico • By a Union Bound, this probability is no more than  $P(X_1 \ge k \log n) + P(X_2 \ge k \log n) + \dots + P(X_n \ge k \log n)$ • Which equals  $\frac{n}{n^k} = n^{1-k}$ 3 \_\_\_\_\_ Height of Skip List \_\_\_\_\_ \_\_\_ Outline \_\_\_\_\_ • If we choose k to be, say 10, this probability gets very small as n gets large • Skip List Wrapup • Master Theorem • In particular, the probability of having a skip list of size exceeding  $k \log n$  is o(1)• So we say that the height of the skip list is  $O(\log n)$  with high probability 1 4 \_\_\_\_\_ Search Time \_\_\_\_\_ — Height of Skip List —— • Note that the expected number of "siblings" of a node, x, at any level i is 2

- Why? Because for a node to be a sibling of x at level *i*, it must have failed to advance to the next level
- The first node that advances to the next level ends the possibility of further siblings.
- This is the same as asking expected number of times we need to flip a coin to get a heads.

- Q: What is the probability that a particular key exceeds height  $k \log n$  for some constant k?
- A: If p = 1/2,  $P(X_i \ge k \log n) = (1/2)^{k \log n} = \frac{1}{n^k}$



Where a and b are constants and f(n) is some other function.
The so-called 'Master Theorem' gives us a general method for solving such recurrences f(n) is a simple polynomial.

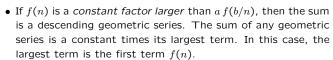
\_ Another View \_\_\_\_\_ • Equivalently, a recursion tree is a complete *a*-ary tree where each node at depth *i* contains the value  $f(n/b^i)$ . • We can now state the Master Theorem • The tree stops when we get to the base case for the recur-• We will state it in a way slightly different from the book rence • Note: The Master Method is just a "short cut" for the re-• We'll assume T(1) = f(1) is the base case cursion tree method. It is less powerful than recursion trees. • Then there are  $\log_b n$  levels to the recursion tree 12 15 Recursion Tree \_\_\_ Master Method \_\_\_\_\_ The recurrence T(n) = aT(n/b) + f(n) can be solved as follows: • For this tree, T(n) is just the sum of all values stored in all levels of the tree • If  $a f(n/b) \leq f(n)/K$  for some constant K > 1, then T(n) = $T(n) = f(n) + a f(n/b) + a^2 f(n/b^2) + \dots + a^i f(n/b^i) + \dots + a^L f(n/b^L)$  $\Theta(f(n)).$ • Where  $L = \log_b n$  is the depth of the tree • If  $a f(n/b) \ge K f(n)$  for some constant K > 1, then T(n) =

• Since  $f(1) = \Theta(1)$ , the last term of this summation is  $\Theta(a^L) =$  $\Theta(a^{\log_b n}) = \Theta(n^{\log_b a})$ 

 $\Theta(n^{\log_b a}).$ 

• If a f(n/b) = f(n), then  $T(n) = \Theta(f(n) \log_b n)$ .

\_\_\_ Proof \_\_\_\_



- If f(n) is a constant factor smaller than a f(b/n), then the sum is an ascending geometric series. The sum of any geometric series is a constant times its largest term. In this case, this is the last term, which by our earlier argument is  $\Theta(n^{\log_b a}).$
- Finally, if a f(b/n) = f(n), then each of the L terms in the summation is equal to f(n).

• It's not hard to see that  $a^{\log_b n} = n^{\log_b a}$ 

$a^{\log_b n}$	=	$n^{\log_b a}$	(2)
$a^{\log_b n}$	=	$a^{\log_a n * \log_b a}$	(3)

- (3)  $\log_b n = \log_a n * \log_b a$ (4)
- We get to the last eqn by taking log<sub>a</sub> of both sides
- The last eqn is true by our third basic log fact

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\_\_\_\_ A "Log Fact" Aside \_\_\_\_\_

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\_\_\_ Example \_\_\_\_

- T(n) = T(3n/4) + n
- If we write this as T(n) = aT(n/b) + f(n), then a = 1, b = 4/3, f(n) = n
- Here a f(n/b) = 3n/4 is smaller than f(n) = n by a factor of 4/3, so  $T(n) = \Theta(n)$
- $T(n) = T(n/2) + n \log n$
- If we write this as T(n) = aT(n/b) + f(n), then  $a = 1, b = 2, f(n) = n \log n$
- Here  $a f(n/b) = n/2 \log n/2$  is smaller than  $f(n) = n \log n$  by a constant factor, so  $T(n) = \Theta(n \log n)$

