Shared Memory in an Adverse Environment

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Shared Memory: An Outsiders View

- Abstracts out messy details about communication thereby allowing simpler description of distributed algorithms
- Decouples 1) development of distributed algorithms and 2) memory management

Wireless Networks with Jamming

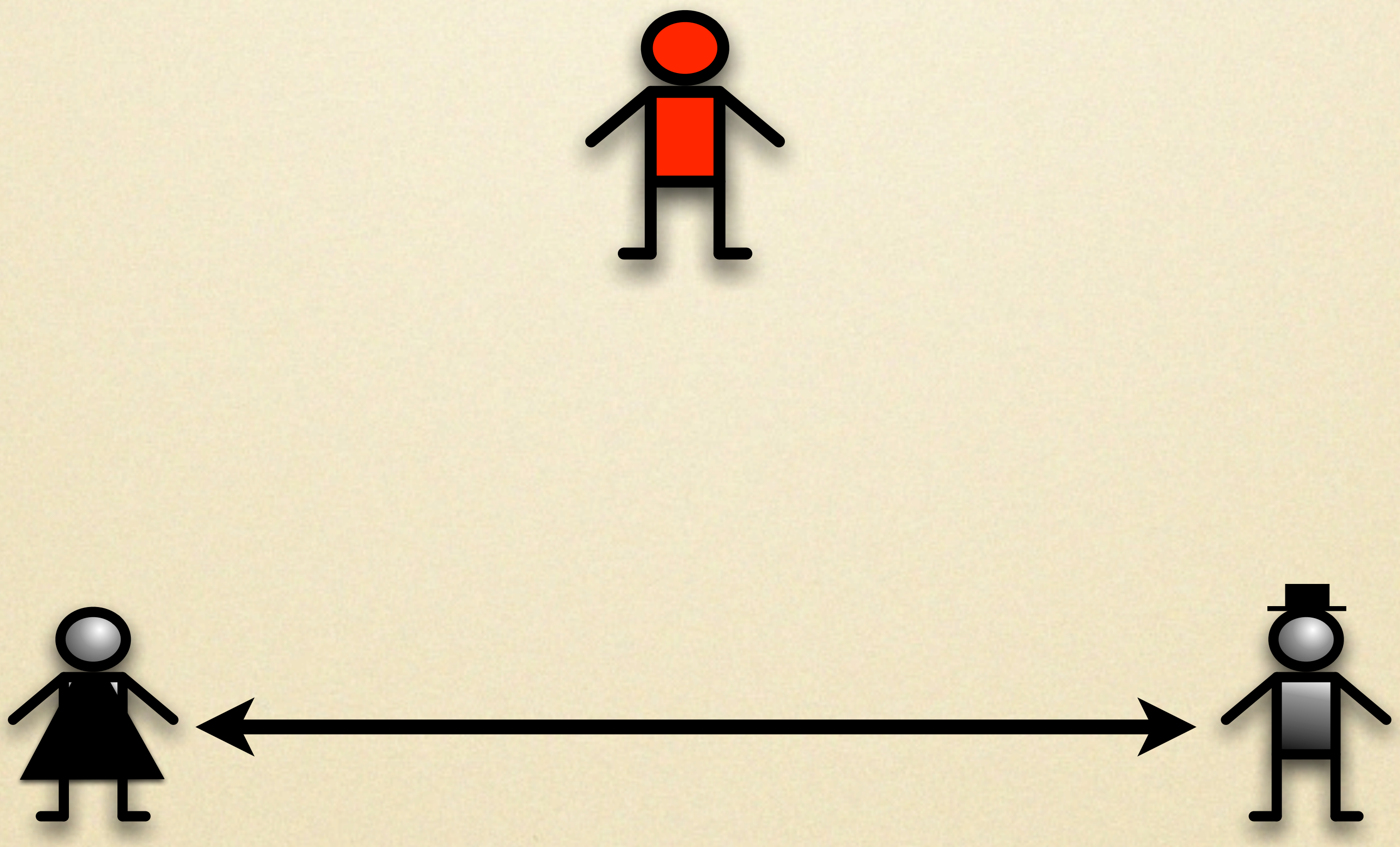
- Synchronous communication; time divided into slots
- All players commit to actions simultaneously
- Easy: Adversary can jam only; Hard: Adversary can spoof

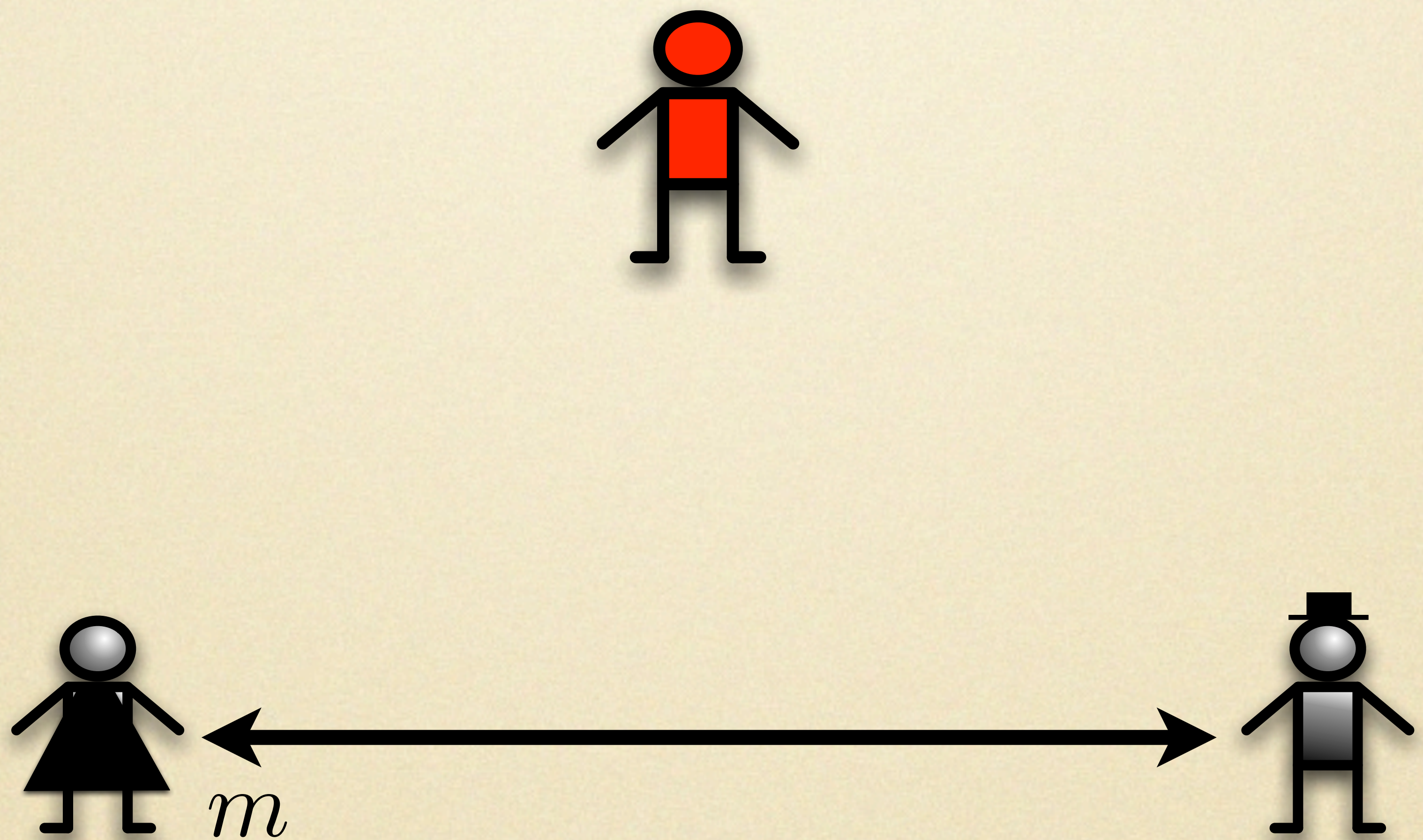
Our Communication Model: Jammable Channel

- Alice has a message she wants to send to Bob
- Adversary doesn’t know random bits of Alice or Bob
- Each player pays a cost (mW) for each action (send, listen, or jam)
- Adversary has a finite but unknown budget, B

Goal

- Simulate a Single Writer, Multiple Reader (SWMR) shared register

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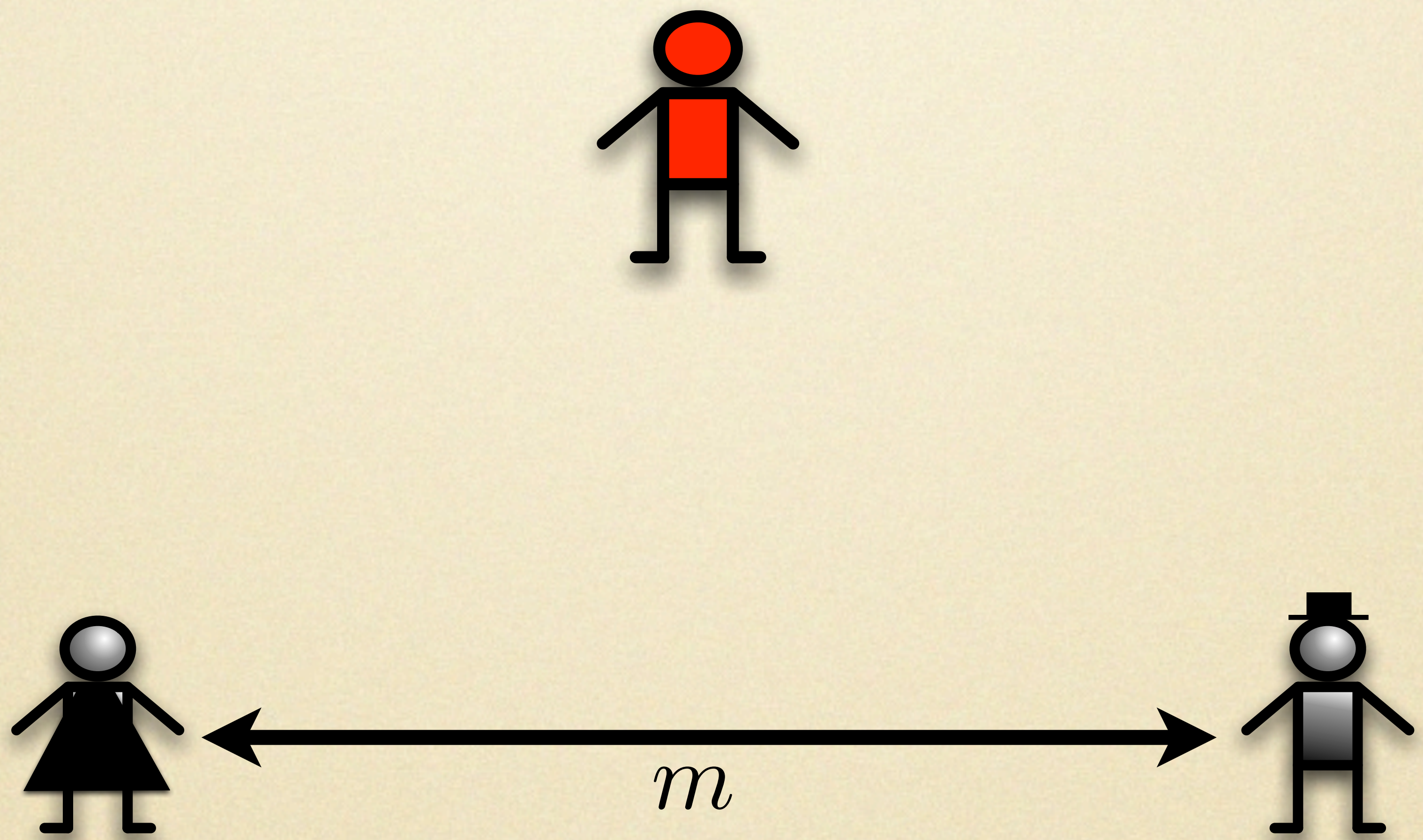
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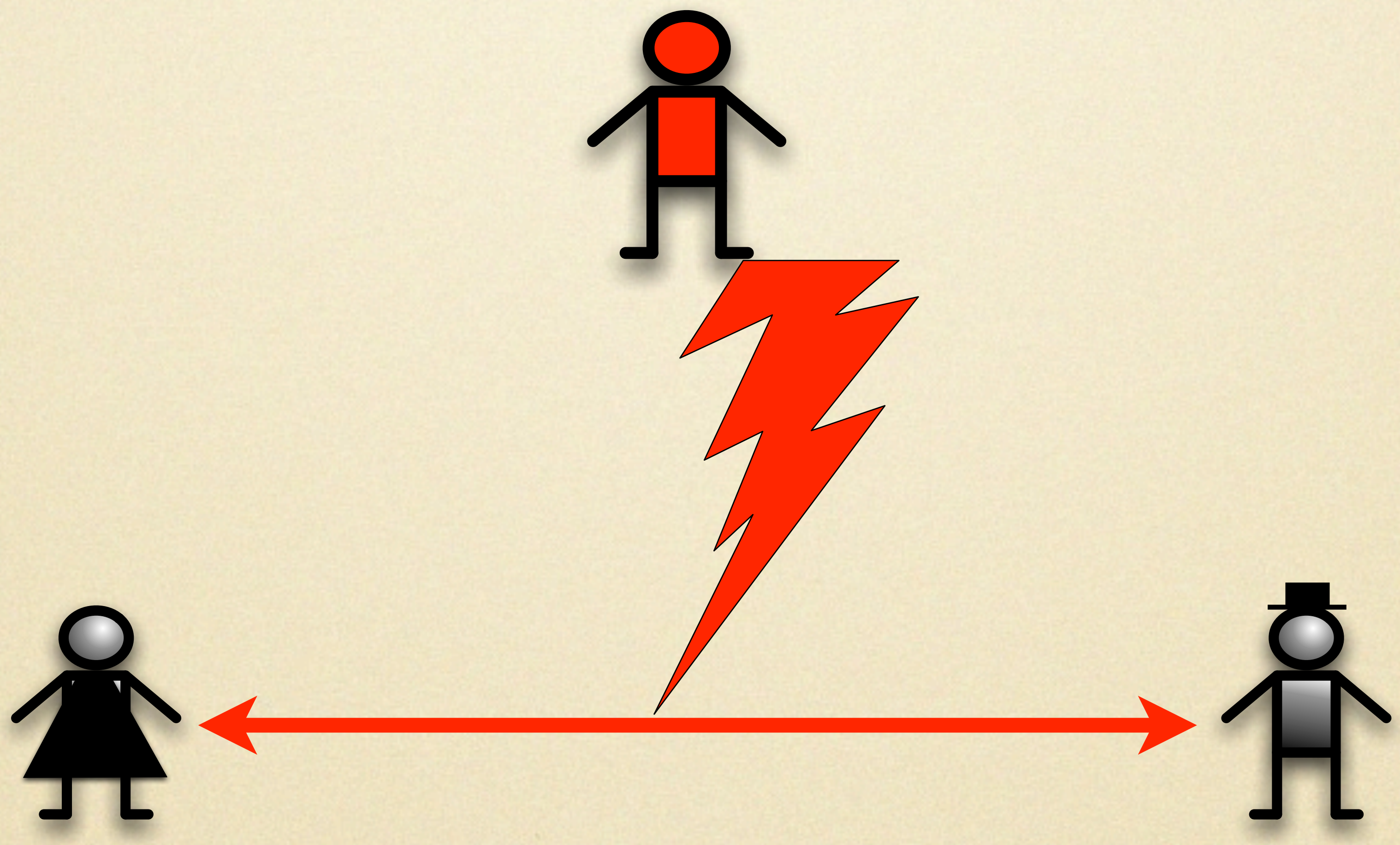

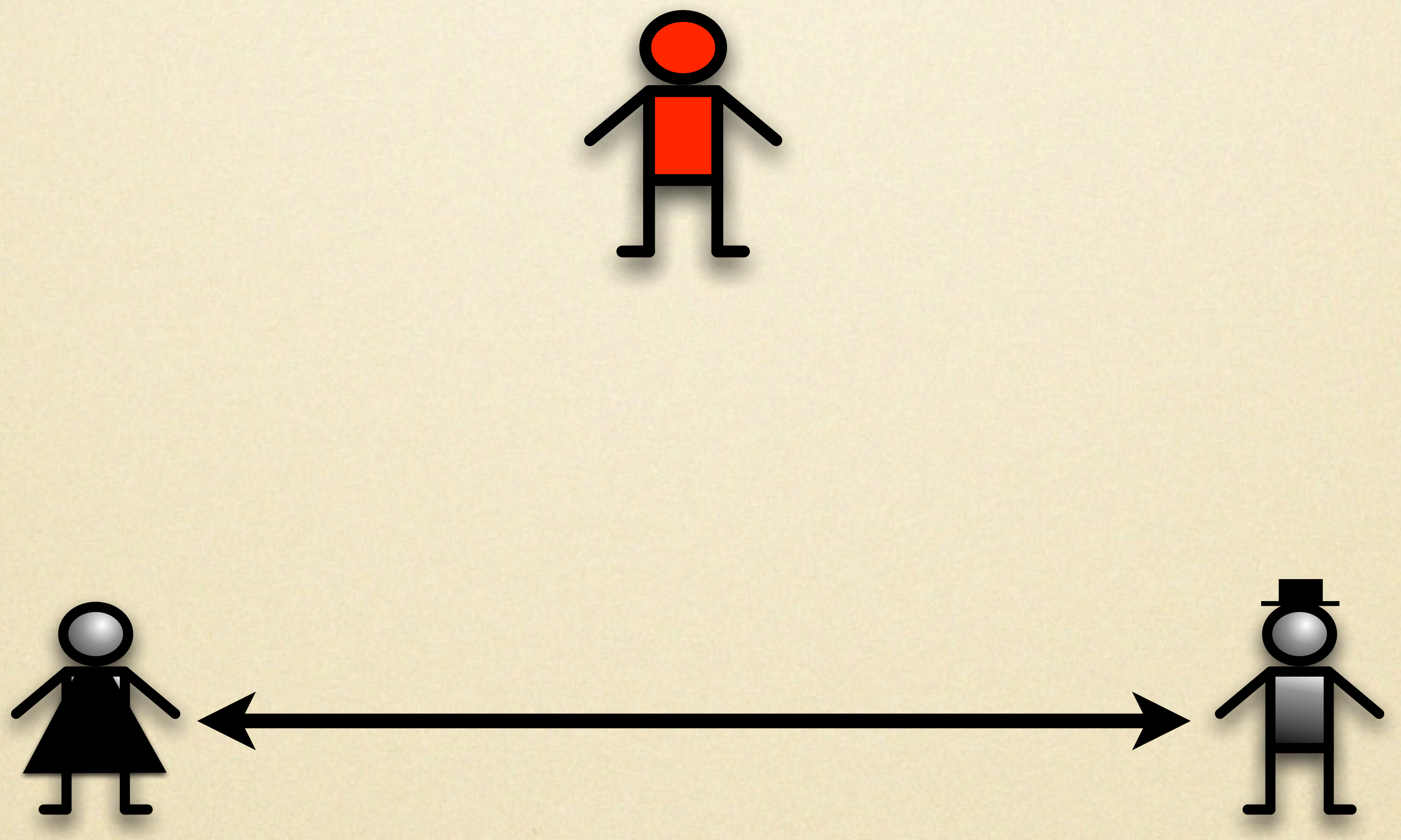

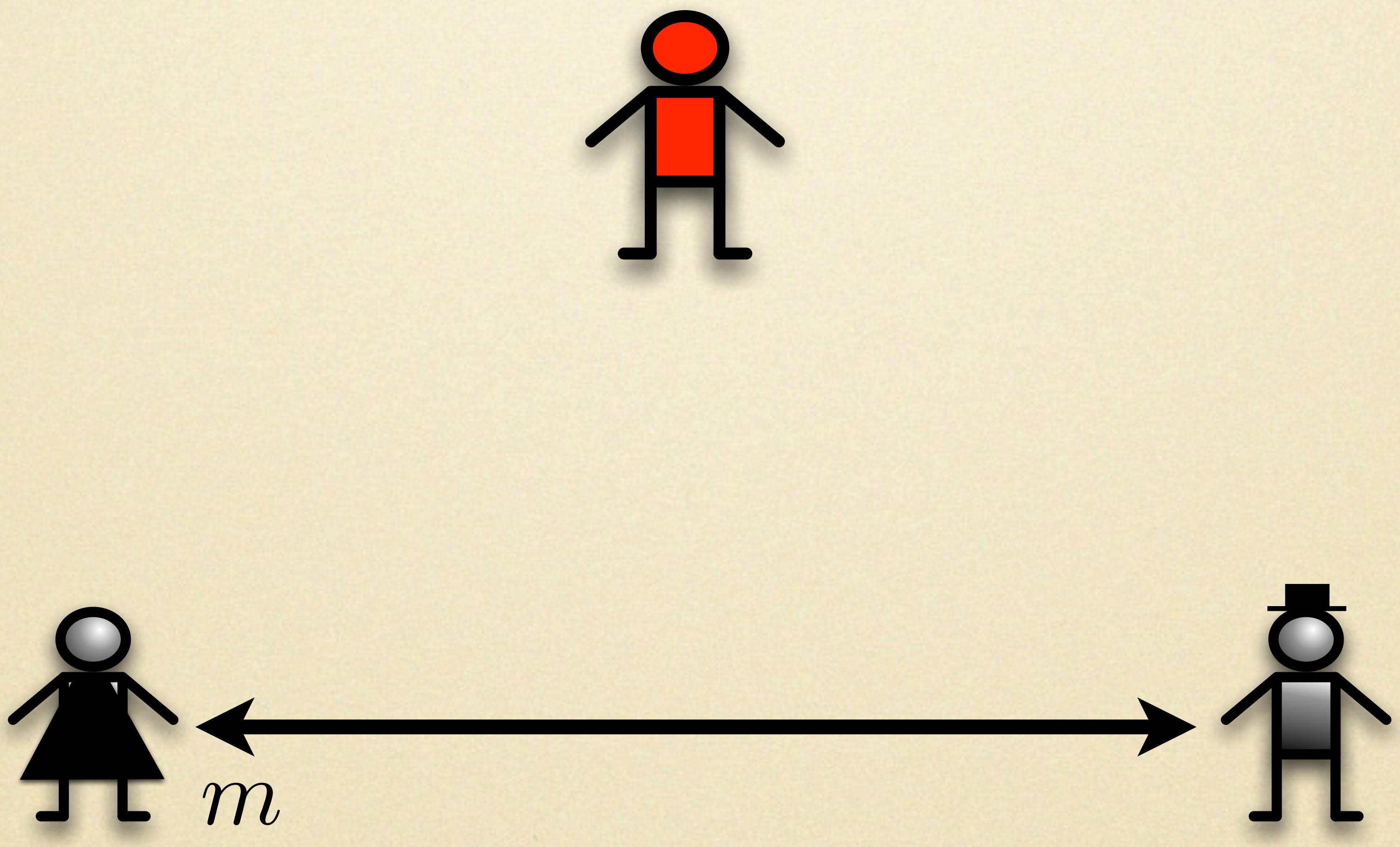
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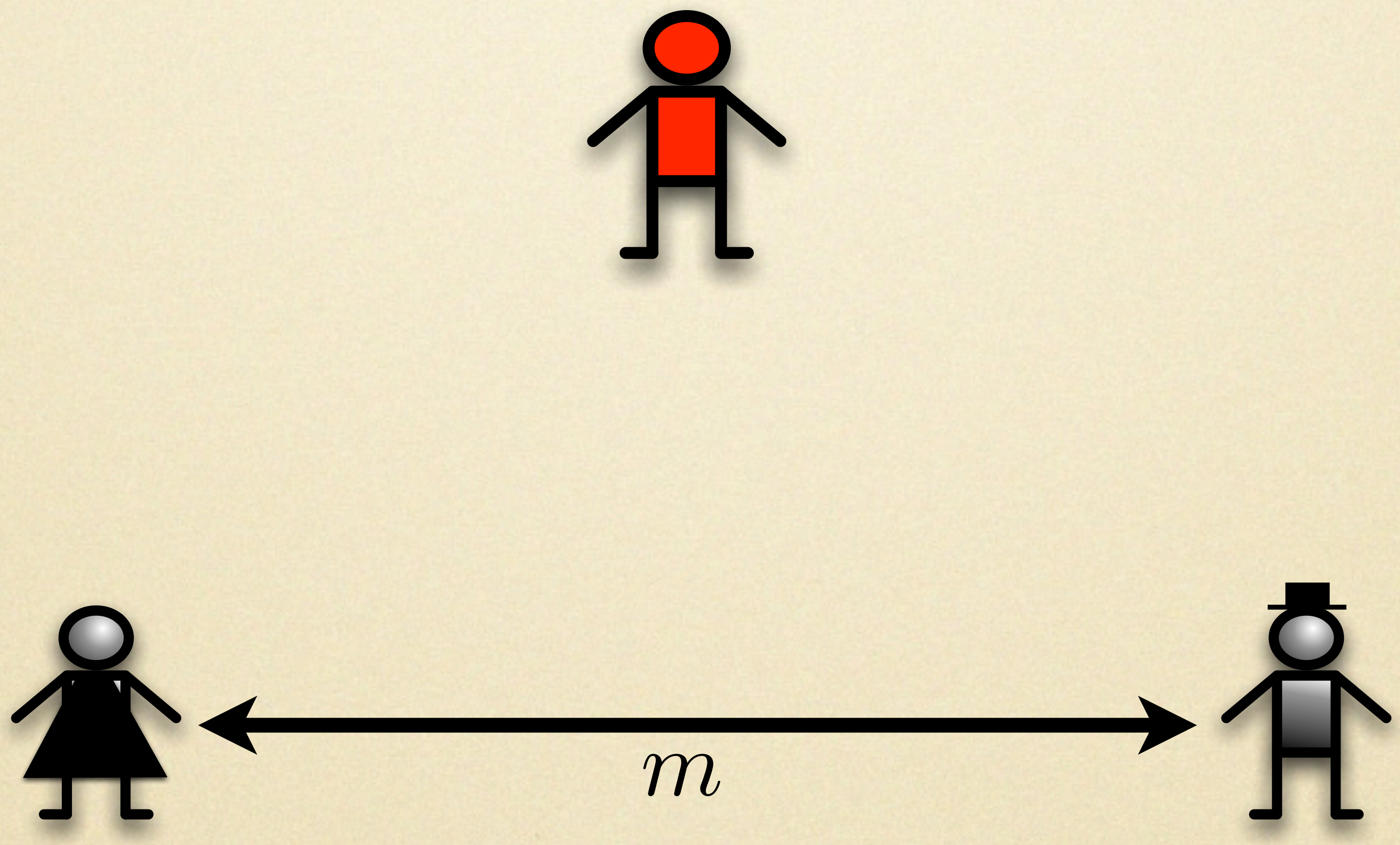
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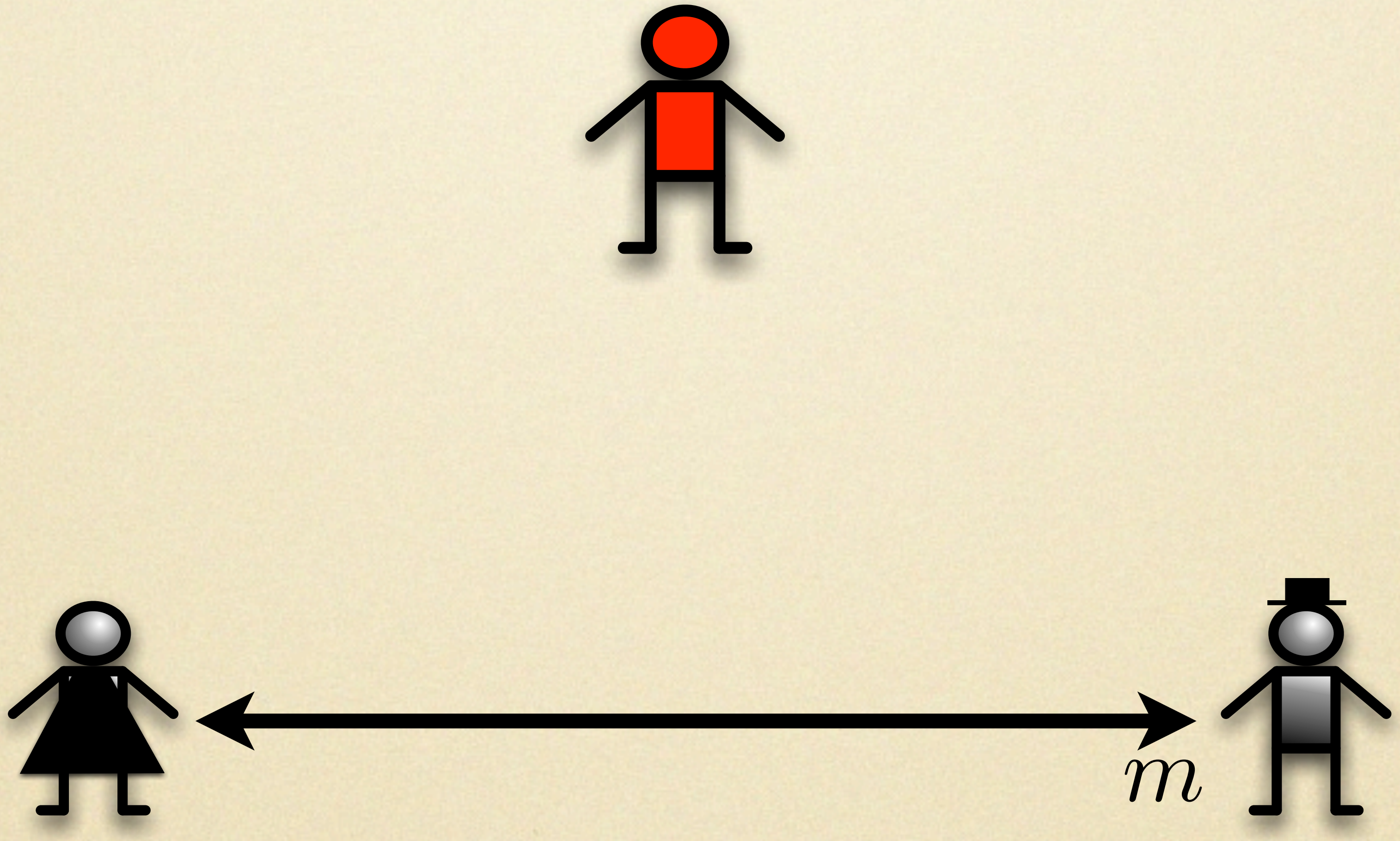
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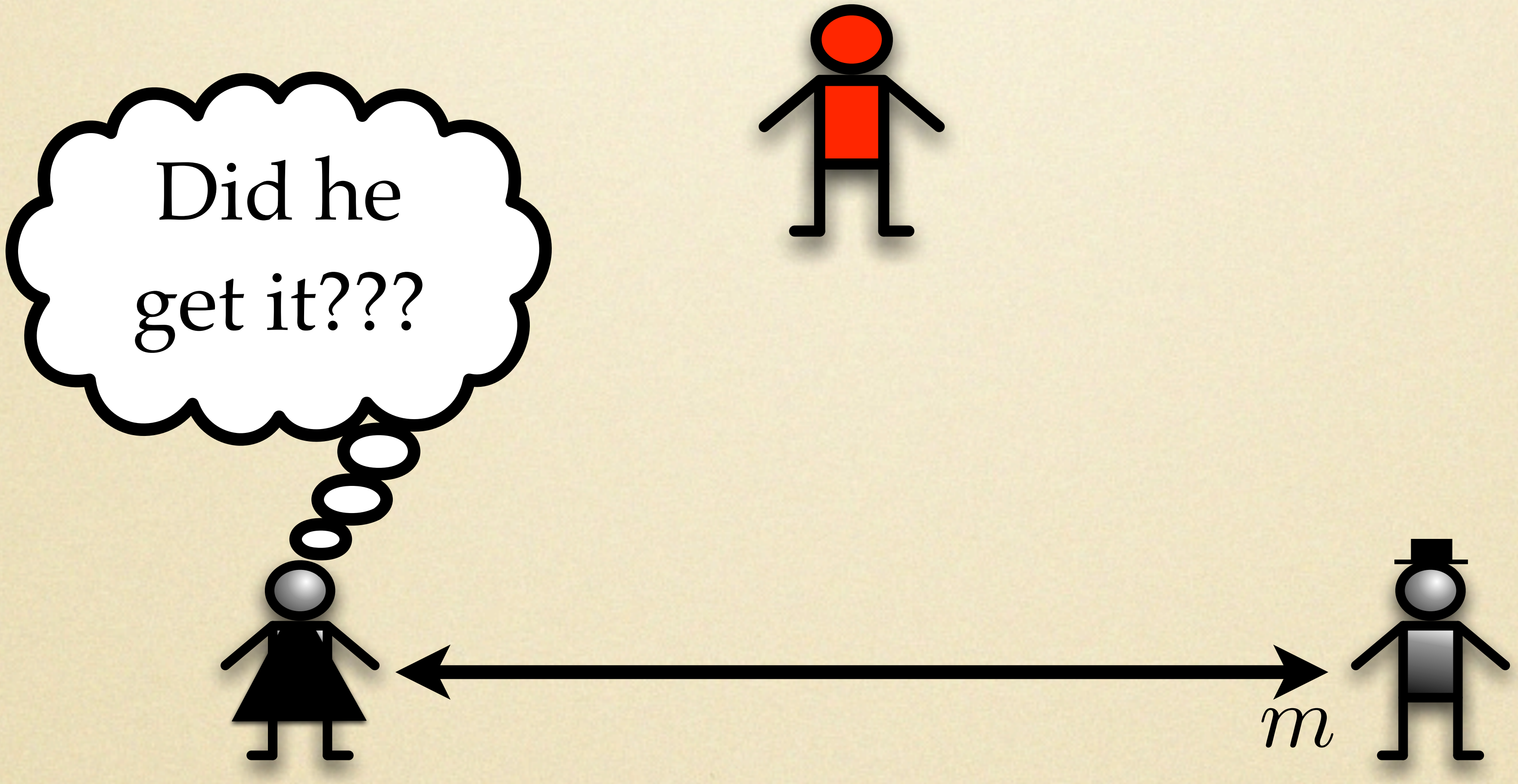
Did he get it???
m

Costs

- Costs \$S to send on channel
- Costs \$L to listen on channel
- Costs \$J to block channel
- Adv. spends \$B

Costs - Sensors

- Costs $S to send on channel 38mW
- Costs $L to listen on channel 35mW
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We assume S, L and J are O(1) and B is unknown but finite

Key Assumption

- If Alice or Bob listen on channel when Adv. jams it, they can detect a “collision”

An Idea

- A round consists of n slots
- Alice sends w/ prob $c/\sqrt{n}$
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After each failed round, n can double in size

Result

- If adversary can only jam, the previous algorithm creates a secure (but randomized) communication channel between Alice and Bob
- Cost of sending and ACK of a message in this channel is: $\tilde{O}(B^{1/2} + 1)$
- Note: Bob must know when to listen to determine if Alice is writing (seems unavoidable)

SWMR Register

- Idea: create such a channel between the writer for the register and each "server", and each reader of the register and each server.
- Using standard tricks (e.g. [ABD '89]) can implement a SWMR shared Register that tolerates fail-stop faults on the servers.

Problems

- Problem 1: What if the adversary can spoof in addition to jamming?
- Problem 2: What if there is only one channel for everyone?

Problem 1: Spoofing

- Now imagine Adv. can spoof Bob (but not Alice)
- Idea: Alice stops sending only if she hears a silent slot
- Problem: Adv. can keep sending fake requests and thereby bankrupt Alice
- Idea: Impose a larger cost to trigger a resend, to mitigate increased cost to Alice

Our Algorithm: Round i

Send Phase: For 2^{ci} slots do

- Alice sends with prob. $2/2^i$
- Bob listens with prob. $2/2^{(c-1)i}$

Req Phase: For 2^i slots do

- If Bob has not received m, Bob sends **req** message
- Alice listens with prob. $4/2^i$

If Alice listened in Req phase and detected no **req** message or collision then algorithm terminates

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Analysis shows it's best to set $c = \varphi$

Result

Theorem: Our algorithm has the following properties:

- The expected cost to Alice and Bob is $O(B^{\varphi-1} + 1) = O(B^{0.62} + 1)$.
- Alice and Bob terminate within $O(B^{\varphi})$ slots in expectation.

A note on c

- $c > 1$ since otherwise Bob listens with prob > 1
- $c < 2$ since otherwise, adversary can cause Alice to spend more than itself by causing repeated Req failures via jamming the entire Req phase.

Failure

- There *are* two ways that a stage can fail
  - ***Send Failure***: Bob did not get the message in Send phase
  - ***Req Failure:*** Alice never listened to a silent slot in the Req phase. Note: this type of attack only makes sense after Bob terminates.

Jamming

- We call a round *send-jamming* if the adversary jams at least half the slots in the send phase
- We call a round *req-jamming* if the adversary jams at least half the slots in the req phase

Lemma 1

Lemma 1. Consider a round that is not send-jamming. The probability that Bob does not receive the message from Alice is less than e^{-2}.

Lemma 1 proof

- Let $s = 2^{ci}$ be the number of slots in the Send Phase
- Let p_A be the probability that Player A sends in a particular slot. Let p_B be the probability that Player B listens in a particular slot.
- Let $X_j = 1$ if the message is not delivered from Player A to Player B in the j^{th} slot.

Lemma 1 Pf (Cont'd)

- Let $q_i = 1$ if adversary does not jam slot i; 0 otherwise
- Then $Pr[X_i = 1 \mid X_1 X_2 \cdots X_{i-1} = 1] = 1 - p_A p_B q_i$
- Thus $Pr[$ m not delivered $] = \prod_{j=1}^{s}(1 - p_A p_B q_j) \leq e^{-p_A p_B \sum_{j=1}^{s} q_j} < e^{-2}$
- Since $p_A p_B \sum_{j=1}^{s} q_j > (2/2^i)(2/2^{(c-1)i})(s/2) > 2$

Lemma 2

Lemma 2. *Assume Bob has received m by round i and that round i is non req-jamming. Then the probability that Alice retransmits m in round $i+1$ is less than e^{-2}.*

Proof

- Let $s = 2^i$ be number of slots in the req phase and $p = 4/2^i$ be the probability that Alice listens in a slot.
- For slot j, let $X_j = 1$ if Alice does not terminate and 0 otherwise.
- Let $q_j = 1$ if the adversary does not jam slot j and 0 otherwise
- Then $Pr[X_j = 1] = (1 - pq_j)$.
- Therefore, $Pr[X_1 X_2 \cdots X_s = 1] \leq e^{-p \sum_{j=1}^{s} q_j} < e^{-2}$.

Lemma 3

Lemma 3. Assume that Player B is correct and there are no send-jamming or ack-jamming rounds. Then, the expected cost of each player is $O(1)$.

Proof

- Using Lemma 1, the expected cost to Alice is at most $\sum_{i=2}^{\infty} e^{-2(i-2)} \cdot (2 \cdot 2^{(c-1)i} + 4) \leq \sum_{i=2}^{\infty}(e^{5-i} + 4 \cdot e^{-2(i-1)}) = O(1)$.
- Using Lemma 2, expected cost to Bob is at most $\sum_{i=2}^{\infty} e^{-2(i-2)} \cdot (2^{i+1} + 2^{i}) \leq \sum_{i=2}^{\infty}(e^{5-i} + e^{4-i}) = O(1)$.

Lemma 4

Lemma 4. Assume there is at least one send-jamming round. Then the expected cost to Alice is $O(B^{(c-1)/c} + B^{c-1})$ and the expected cost to Bob is $O(B^{1/c})$

Main Theorem

Theorem: Our algorithm has the following properties:

- The expected cost to Alice and Bob is $O(B^{\varphi-1}+1) = O(B^{0.62}+1)$.
- Alice and Bob terminate within $O(B^{\varphi})$ slots in expectation.

Proof (Adv. Cost)

- Let i be the last send send-jamming round; $j \geq i$ be the last req-jamming round
- If no such req-jamming round exists $j = 0$
- Then cost to the adversary, B, is $\Omega(2^{ci} + 2^{j})$

Proof (Alice's cost)

- Using Lemma 1, the expected cost to Alice, prior to m being delivered is $O(2^{(c-1)i}) + \sum_{k=1}^{\infty} e^{-2(k-1)}(2 \cdot 2^{(c-1)(i+k)} + 4) = O(2^{(c-1)i})$ since $c < 2$
- Now, using Lemma 2, the expected cost to Alice after delivery is $O(2^{(c-1)j}) + \sum_{k=1}^{\infty} e^{-2(k-1)}(2 \cdot 2^{(c-1)(j+k)} + 4) = O(2^{(c-1)j})$ since $c < 2$
- Therefore, the total expected cost to Alice is $O(2^{(c-1)i} + 2^{(c-1)j})$
- Since $B = \Omega(2^{ci} + 2^{j})$, this cost as a function of B is $O(B^{(c-1)/c} + B^{(c-1)})$

Proof (Bob's Cost)

- Using Lemma 1, Bob's expected cost prior to receiving m is $O(2^i) + \sum_{i=1}^{\infty} e^{-2(k-1)}(2 \cdot 2^{i+k} + 2^{i+k}) = O(2^i)$
- Thus, the expected cost for Bob as a function of B is $O(B^{1/c})$

Proof

- By Lemma 4, the expected cost of Alice is $O(B^{(c-1)/c} + B^{(c-1)}))$ and the expected cost of Bob is $O(B^{1/c})$
- Therefore, the exponents that control the cost ratios are $(c-1)/c, c-1, 1/c$
- Since $1 < c < 2$, we know that $1/c > (c-1)/c$. Thus we solve for c in $c - 1 = 1/c$
- This gives $c = (1 + \sqrt{5})/2$

Notes

- What if the adversary tries to spoof Bob?
  - The theorem still holds (analysis omitted)
- However Alice's messages **must** be authenticated with e.g. digital signature.
  - Seems inherent requirement

Problem 2

- What if there is only one channel?
- Want Alice to be able to communicate to multiple Bob's simultaneously

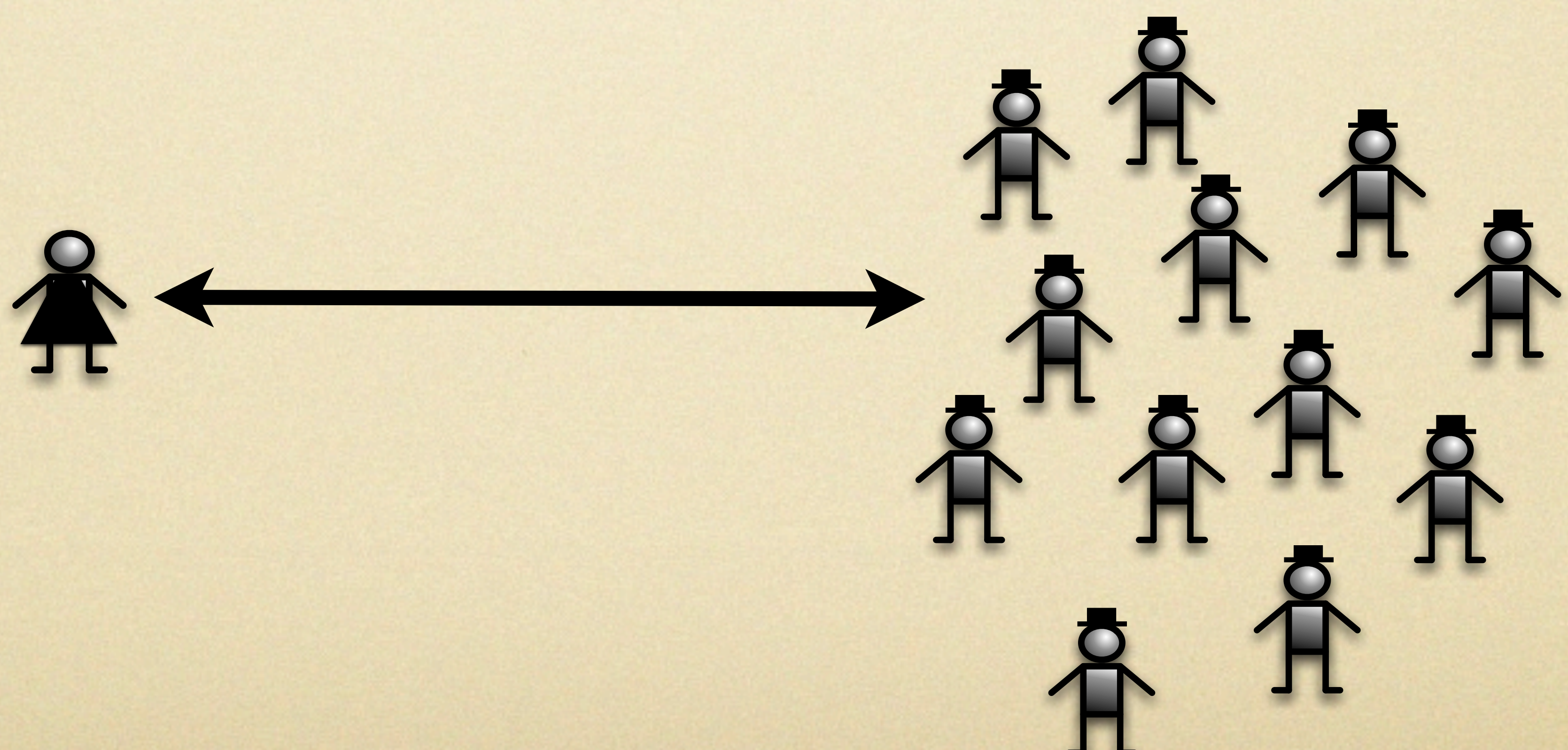
Model

- In a given time slot, when a single player sends a message on the channel, all listening players hear the message
- In a given time slot, when more than one players sends a message, all listening players hear a jam

Old Result

Theorem: There exists an algorithm for one sender and n receivers that ensures the message is delivered to all receivers and has the following costs:

- The sender's expected cost is $O(B^{\varphi-1}\log n + \log^{\varphi} n)$
- The expected cost to any receiver is $O(B^{\varphi-1} + \log n)$
- The worst case number of slots used is $O((B + \log^{\varphi-1})^{\varphi+1})$



New Conjecture

- There exists an algorithm for one sender and n receivers such that
- The message is received and acknowledged whp
- All players terminate with expected cost:

  $O((B/n + 1)\log^2(B + n))$

Candidate Algorithm

For $i = 0$ to ∞

 Repeat i times:

  Set $S \leftarrow 4$

  Repeat for 2^i slots

   Send the message or noise if the message is not known with probability $S/2^i$.

   Listen with probability $iS/2^i$

  CASE:

  a) if a message is heard, then

   with probability 1/2: terminate; else $S \leftarrow 2S$ for the next repetition

  b) if mostly empty slots are heard $S \leftarrow 2S$ for the next repetition

  c) if jamming or collision is heard $S \leftarrow S$ for the next repetition

Problems

- Assume we can enable broadcast and acknowledgement in $\tilde{O}(B/n + 1)$
- Want: no cost to anyone when the register doesn't change. Seems hard.
- Q: Can we allow that there is only a "small" cost in rounds where the value of the register doesn't change?

Adapt to DOS Attacks

- Consider a p2p system with a shared memory
- Are send, listen and jam costs similar?
- Estimates based on Amazon.com's EC2
  - $.17 to send 1GB
  - $.9 to "listen" to 1GB (i.e. process)

What is a slot?

- a slice of time
- a unique communication channel
  - physical channel: wireless nodes broadcast at different frequencies
  - virtual channel: messages indexed by a channel id

Problems

- Connections to known shared memory results? Are there lower/upper bounds from the message passing model that apply to this model with jamming?
- Can we bound the competitive ratio achievable even if our algorithm is randomized?

Questions