Species interaction in a toy ecosystem

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Abstract

In this paper, I construct a model to describe the interactions in a toy ecosystem consisting of five species showing various predatory and competition interactions. I tried to analyse the system using logistic equation and the allee effect. However, to account for the interactions, I had to add suitable modifications to these equations to achieve reasonable results. Since, in a predatory system, a species itself is the resource, the predator cannot survive without its prey. This had to be accounted for. Also, predators have different degree of affections for their different preys. This was also accounted for. I tried to test the system for sensitivity to the number of species in a system and to try to identify critical or sensitive species. The results were along predictable lines with the low hierarchy species being critical for others survival.

1 Introduction

In an ecosystem, there can be many relations between species. Here, I set up a small ecosystem with the following relationships: direct food chain, indirect mutualism, exploitative competition and apparent competition. This ecosystem has five interacting species: A,B,C and D. It is depicted in figure 1.

There are four main interactions within this system:

- *Direct food chain:* A is a predator of B, and B of C, as shown in figure 2.
- *Exploitative Competition* Both B and C exploit E, as in figure 3.
- *Apparent Competition* B exploits both D and E, as in figure 4.
- *Indirect Mutualism*

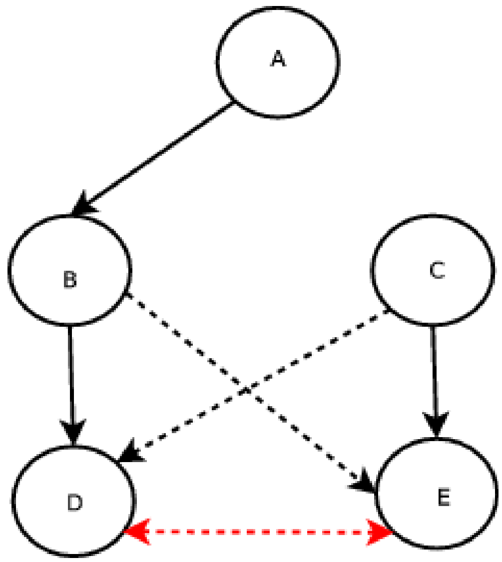

Figure 1: Toy Ecosystem

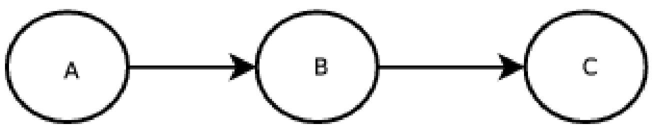

Figure 2: Direct Food Chain

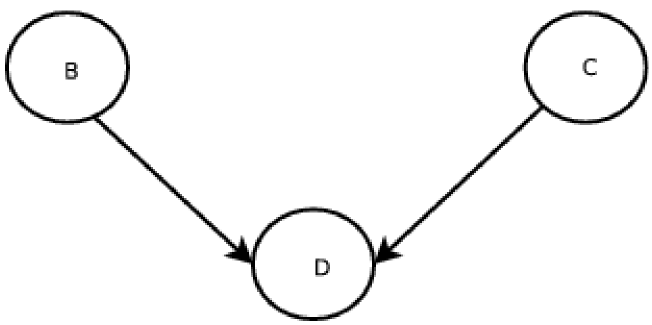

Figure 3: Exploitative Competition

B mainly exploits A and secondarily exploits D, whereas E mainly exploits D and secondarily exploits A, thus, relating A and D. This is shown in figure 5.

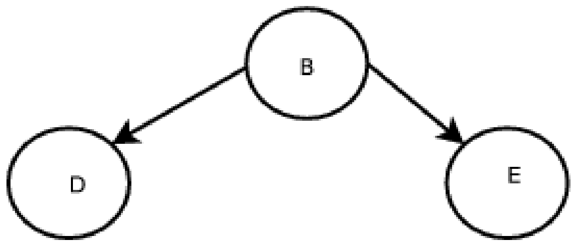

Figure 4: Apparent Competition

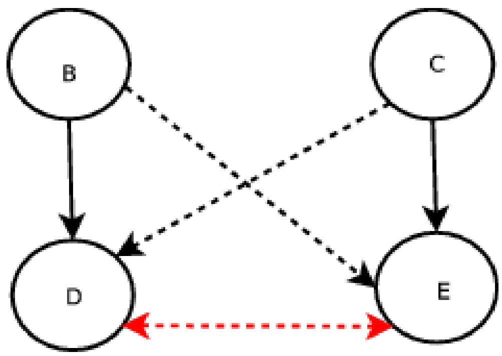

Figure 5: Indirect Mutualism

2 Logistic Equation

The first attempt was by constructing a set of coupled differential equations based on logistic equations. These are in the form:

$$
\begin{aligned}
\frac{dn_A}{dt} &= a_1 n_A - b_1 {n_A}^2 + c_1 n_A n_B \\
\frac{dn_B}{dt} &= a_2 n_B - b_2 {n_B}^2 - c_1 n_A n_B + d_2 n_B n_D + d_2 r n_B n_E \\
\frac{dn_C}{dt} &= a_3 n_C - b_3 {n_C}^2 + c_3 n_C n_E + c_3 r n_C n_D \\
\frac{dn_D}{dt} &= a_4 n_D - b_4 {n_D}^2 - d_2 n_B n_D - c_3 r n_C n_D \\
\frac{dn_E}{dt} &= a_5 n_E - b_5 {n_E}^2 - d_2 r n_B n_E - c_3 n_C n_E
\end{aligned}
$$

Here the term r expresses the relationship of indirect mutualism. It is

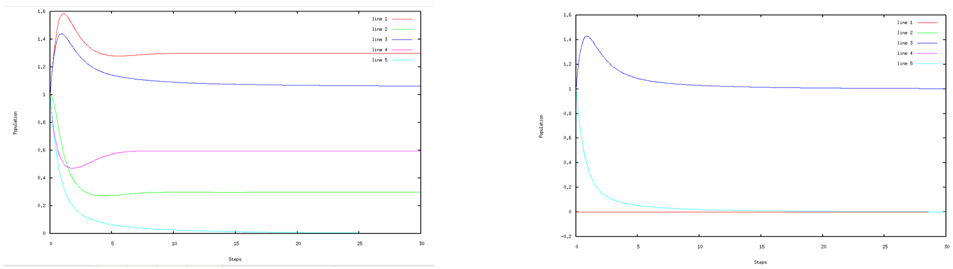

Figure 6: a) The logistic based equation for 30 steps. b) The system in absence of A (The Lines overlap)

the fraction of predation that a predator does on the secondary prey as compared to its primary prey. I took the value $r = 0.1$ for my simulations.

Fixed Points

The logistic equation has an unstable fixed point at $n = 0$ and a stable fixed point of a positive value. Thus, following are the fixed points, which are not resloved in the form of the parameters of the equations:

$$
\begin{aligned}
n_A^* &= 0, \frac{a_1 + c_1 n_B}{b_1} \\
n_B^* &= 0, \frac{a_2 - c_1 n_A + d_2(n_D + rN_E)}{b_2} \\
n_C^* &= 0, \frac{a_3 + c_3(n_E + rN_D)}{b_3} \\
n_D^* &= 0, \frac{a_4 - d_2 n_B - c_3 r N_C}{b_4} \\
n_E^* &= 0, \frac{a_5 - d_2 r n_B - c_3 N_C}{b_5}
\end{aligned}
$$

Throughout the rest of the discussion, however, I would be giving the results obtained by using the ODE solver *lsode* in *Octave*.

Analysis

The population of B seems surprisingly less and sensitive to conditions, but this is due to its exploitation by the top-level A. Importantly, B is the food

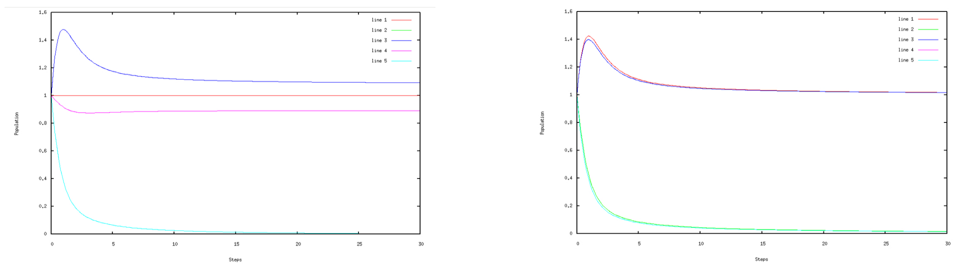

Figure 7: a) The Logistic system in absence of B. b) The system in absence of D

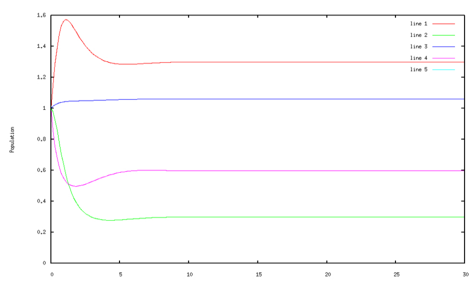

Figure 8: The Logistic system in absence of E

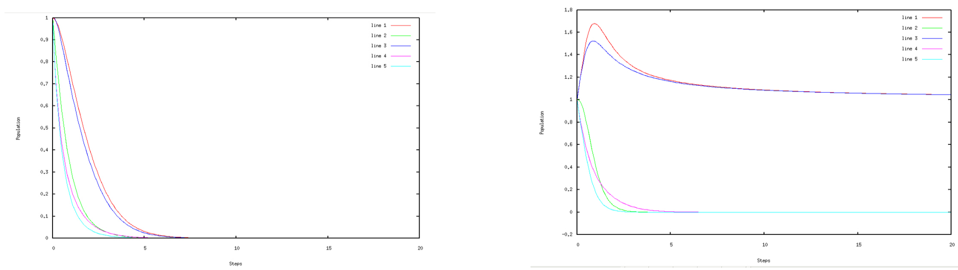

Figure 9: a) The cubic equation for all parameters 1. b) $a_i = 2$ for all i

resource of A and D and E of B and C, yet the populations do not disappear when these go to zero. Thus, this model is not appropriate.

3 Incorporating the Allee effect

The problem with the logistic equation based system is the absence of a stable fixed point at zero, not capturing the idea that very low populations have a smaller chance of survival. To incorporate this idea, we try to use a cubic equation for the system.

1. *A cubic effect*

$$
\begin{aligned}
\frac{dn_A}{dt} &= a_1 {n_A}^2 - b_1 {n_A}^3 - l_1 n_A + c_1 n_A n_B \\
\frac{dn_B}{dt} &= a_2 {n_B}^2 - b_2 {n_B}^3 - l_2 n_B - c_1 n_A n_B + d_2 n_B n_D + d_2 r n_B n_E \\
\frac{dn_C}{dt} &= a_3 {n_C}^2 - b_3 {n_C}^3 - l_3 n_C - c_3 n_C n_E + c_3 r n_C n_D \\
\frac{dn_D}{dt} &= a_4 {n_D}^2 - b_4 {n_D}^3 - l_4 n_D - d_2 n_B n_D - c_3 r n_C n_D \\
\frac{dn_E}{dt} &= a_5 {n_E}^2 - b_5 {n_E}^3 - l_5 n_E - d_2 r n_B n_E - c_3 n_C n_E
\end{aligned}
$$

2. *The Allee effect* The Allee effect tries to capture the condition that the maximum growth is near the middle level population of species. If there are too few individuals, they may have difficulty finding mates, if too many, they have too much competition. This set of equations captures this effect better:

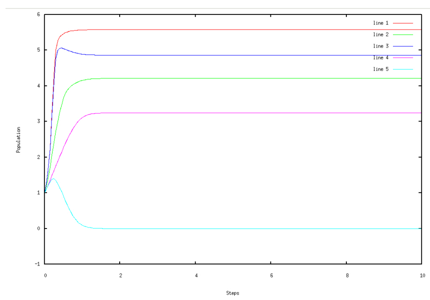

Figure 10: $a_i = 5$ for all i

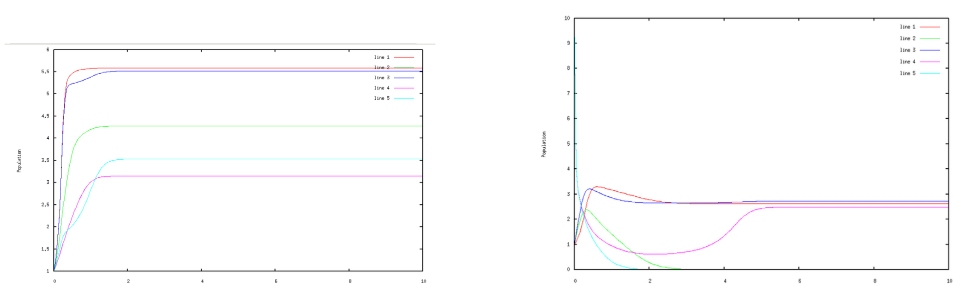

Figure 11: a) $a_i = 5$ for all i except $a_5 = 5.5$, b) D,E with initial population of 10.

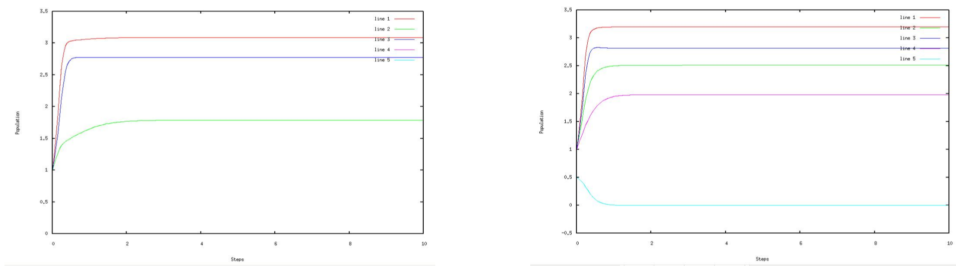

Figure 12: a) Allee effect Population without D and E. b) E with population 0.5

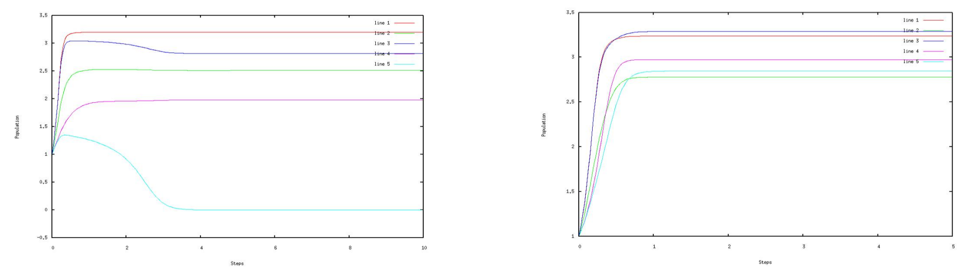

Figure 13: a)Allee: $b_i = 2$ for all i, $a_4 = a_5 = 1.5$, b)$b_i = 2$ for all i, $a_4 = a_5 = 2$

$$
\begin{aligned}
\frac{dn_A}{dt} &= 2a_1b_1{n_A}^2 - b_1{n_A}^3 + (l_1 - a_1^2)n_A + c_1n_An_B \\
\frac{dn_B}{dt} &= 2a_2b_2{n_B}^2 - b_2{n_B}^3 + (l_2 - a_2^2)n_B - c_1n_An_B + d_2n_Bn_D + d_2rn_Bn_E \\
\frac{dn_C}{dt} &= 2a_3b_3{n_C}^2 - b_3{n_C}^3 + (l_3 - a_3^2)n_C - c_3n_Cn_E + c_3rn_Cn_D \\
\frac{dn_D}{dt} &= 2a_4b_4{n_D}^2 - b_4{n_D}^3 + (l_4 - a_4^2)n_D - d_2n_Bn_D - c_3rn_Cn_D \\
\frac{dn_E}{dt} &= 2a_5b_5{n_E}^2 - b_5{n_E}^3 + (l_5 - a_5)n_E - d_2rn_Bn_E - c_3n_Cn_E
\end{aligned}
$$

Analysis

The population now goes to zero if there are too few individuals, however, the problem of foodchain persists. B is the food resource of A

and D and E of B and C, yet the populations do not disappear when these go to zero. Thus, this model is not appropriate.

4 Our new model

To overcome the problem with the models discussed above, we have to make sure that the role of the species as food is acknowledged. Thus, we have to add terms such that the species population goes to zero if its food i.e. prey species does so.

We conceive the following model (we call this effect *Wami effect*! - just for fun, naming after the colloborators):

$$
\begin{aligned}
\frac{dn_A}{dt} &= (2a_1b_1{n_A}^2 - b_1{n_A}^3)(\frac{n_B}{n_B + 0.01}) + (l_1 - a_1^2)n_A + c_1n_An_B \\
\frac{dn_B}{dt} &= (2a_2b_2{n_B}^2 - b_2{n_B}^3)(\frac{n_D + n_E}{n_D + n_E + 0.01}) + (l_2 - a_2^2)n_B \\
& \quad -c_1n_An_B + d_2(1 - r_2)n_Bn_D + d_2r_2n_Bn_E \\
\frac{dn_C}{dt} &= (2a_3b_3{n_C}^2 - b_3{n_C}^3)(\frac{n_D + n_E}{n_D + n_E + 0.01}) + (l_3 - a_3^2)n_C \\
& \quad -c_3(1 - r_3)n_Cn_E + c_3r_3n_Cn_D \\
\frac{dn_D}{dt} &= 2a_4b_4{n_D}^2 - b_4{n_D}^3 + (l_4 - a_4^2)n_D - d_2(1 - r_2)n_Bn_D - c_3rn_Cn_D \\
\frac{dn_E}{dt} &= 2a_5b_5{n_E}^2 - b_5{n_E}^3 + (l_5 - a_5)n_E - d_2r_2n_Bn_E - c_3(1 - r_3)n_Cn_E
\end{aligned}
$$

The rational is that since the resources i.e. the food is the species lower in the food chain, the growth and competition is proportional to that. The 0.01 is added to the denominator to prevent division by zero problems. Also, notice the use of r has been adjusted so that the species divides its food eating when indirect mutualism is involved. The effects are shown in the figures.

Analysis

It follows from our model that after the prey disappears the predator cannot survive for long and dies out soon. Thus, the ecosystem can destabilise if the lower level species are removed. However, there may be richer interaction at lower level and thus removal of a larger number of species may be required.

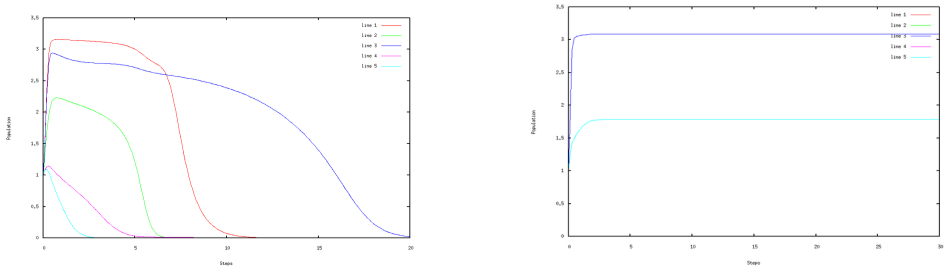

Figure 14: a) Wami effect: Population disappears without food! b) Population without A

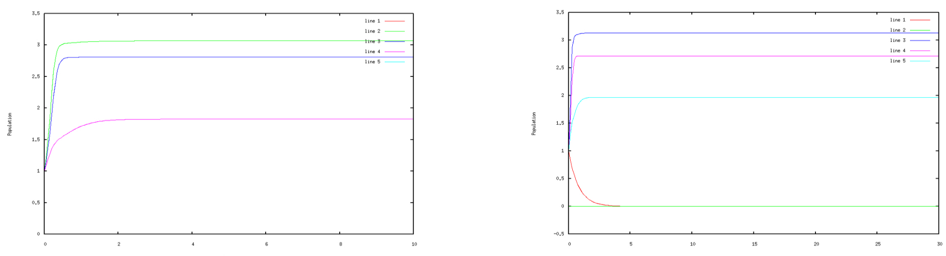

Figure 15: a) Wami! Population without A and E b) Population without B

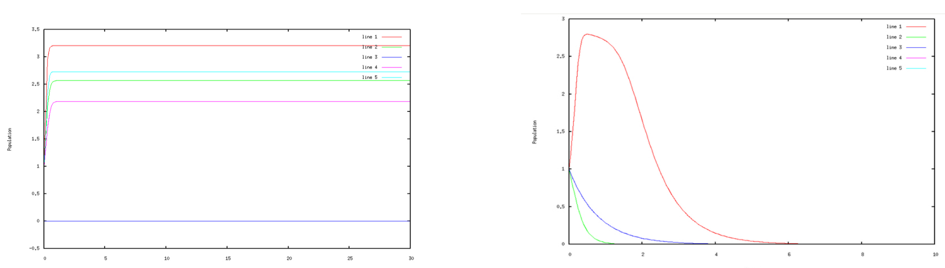

Figure 16: a) Wami! Population without C b) Population without D and E

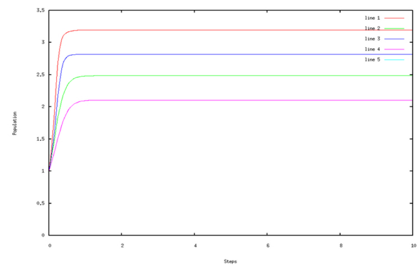

Figure 17: a) Wami! Population without E

Also, the loss of the predators themselves does not seem to make too much of a difference to the stability of the ecosystem. Of course, this model is a gross simplification!

5 An interesting effect

When I used our equation with 0.1 instead of 0.01 in the denominator, I got rather interesting dynamics, which are shown in figure 18. I am not sure if this is a useful illustration though, so I desist its discussion at the moment.

6 Conclusions

This short study gives an idea of the issues involved in modelling interactions among multiple species. The dynamics of species B are interesting in the different models esp. when using the logistic model. Our model helps to conclude that bigger ecosystems generally have good stability and its not easy to destabilise the system by extinction of a small number of species if the interactions are rich enough.

Acknowledgements

I thank *Wenyun Zuo* for suggesting this topic and helping me with details on the ecological aspects.

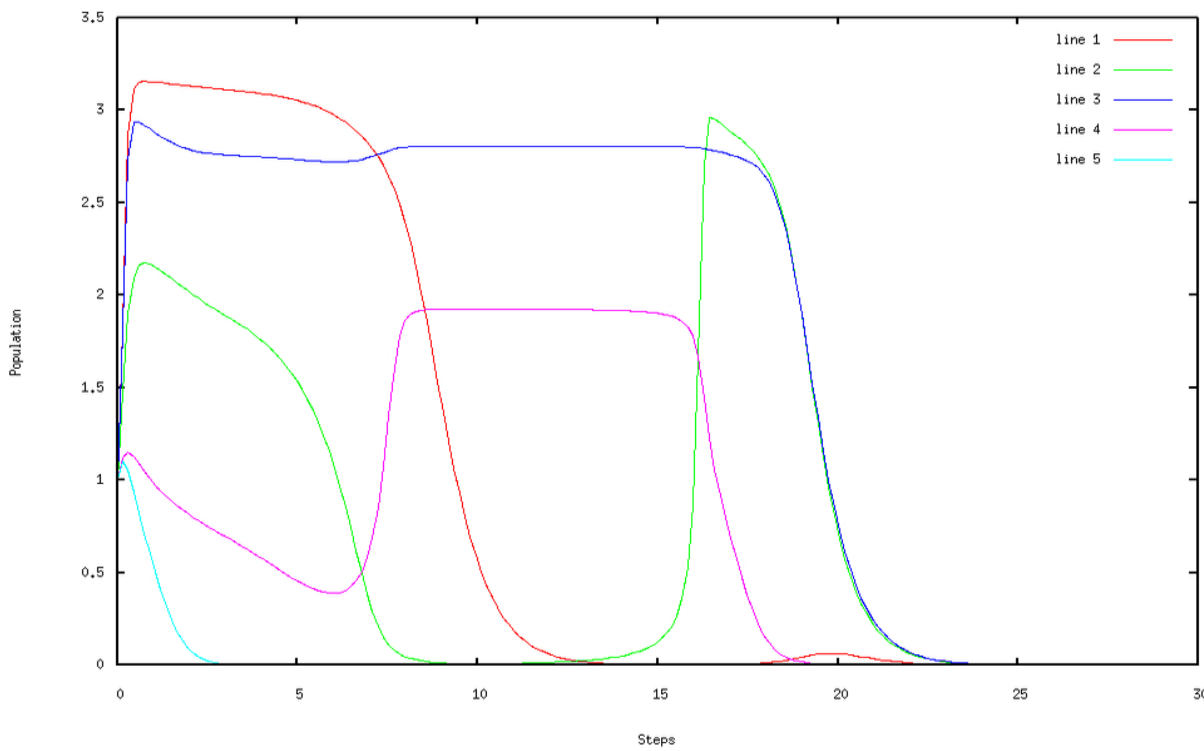

Figure 18: crazy effect!