

Unsupervised Learning of Growing Roadmap in Multi-Goal Motion Planning Problem

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Index Terms—Motion planning, Traveling salesman problem

Abstract—In this paper, we address the multi-goal motion planning problem in which it is required to determine an order of visits of a pre-specified set of goals together with the shortest trajectories connecting the goals. The considered problem is inspired by inspection planning missions, where multiple goals must be visited with a required precision. The problem combines challenges of the combinatorial traveling salesman problem with difficulties of the motion planning. The presented approach is based on unsupervised learning of the self-organizing map technique for the traveling salesman problem applied in the configuration space. This learning technique takes an advantage of acquiring information about exploring the configuration space into a topology of the map that is simultaneously exploited in determination of the multi-goal trajectory and further directions of motion planning roadmap expansion. Presented results indicate that the proposed approach is feasible and it is able to provide a solution of the multi-goal motion planning problem without a priori known sequence of the goals visits.

I. INTRODUCTION

Multi-goal motion planning (MGMP) is a problem that can be found in many robotic scenarios where a mobile robot is requested to visit a set of locations to collect data or take sensor measurements. Examples of such robotic missions are data gathering tasks [1], inspection of complex marine structures [2], industrial applications including spot welding, spray painting and drilling [3], [4] and surveillance missions [5]. The task can be formulated as a variant of the Traveling Salesman Problem (TSP), which is known to be NP-hard [6]. If particular trajectories between the locations are known in advance, a solution of the problem can be found by existing combinatoric techniques for a graph based TSP, e.g., using CONCORDE [7]. However, determination of all trajectories between each pair of the locations is a challenging problem itself, as determination of each individual trajectory is a motion planning problem, which can be computationally demanding, i.e., PSPACE-hard [8] for a polyhedral problem representation.

On the other hand, the sequencing part of the MGMP problem can be determined as a solution of the TSP in

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Fig. 1. PhantomX hexapod executing multi goal trajectory

the Euclidean space. Such an approach can provide a straightforward way to obtain a sequence of the goal visits; however, without considering obstacles and using only direct Euclidean distance between the goals, it does not take into account kinematic constraints of complex robots (e.g., a hexapod walking robot from Fig. 1) and thus it does not provide a direct solution how to control the robot motion to fulfill the mission objective efficiently. Therefore motion planning approaches should be directly considered in the context of the multi-goal motion planning.

Probably the most successful and also computationally efficient motion planners are randomized sampling based techniques such as the Probabilistic Roadmap Method (PRM) [9] and Rapidly-exploring Random Tree (RRT) [10]. They can be used to find the required trajectories between the sequence of the goals. However, it means that for n goals up to n^2 trajectories have to be determined, which can be computationally demanding. Notice, these techniques provide feasible trajectories and using relatively recent the Rapidly-exploring Random Graph (RRG) technique [11] to find asymptotically optimal solutions can be even more demanding.

The computational difficulty motivates researchers to study techniques how to avoid computation of all trajectories between the goals. For example in [12], authors consider approximation of the distance cost between the goals as the Euclidean distance that is used for solution of the TSP as a minimum spanning tree. Then, such costs are iteratively updated using a motion planner and the TSP is repeatedly solved until all trajectories in the solution are determined by the motion planner. Authors reported computational reduction in the selected scenarios, but all trajectories between each goal-goal pair may need to be determined in the worst case.

In this paper, we do not assume a planner providing goal-goal trajectories like in [12] and we rather focus on a different approach to the MGMP that is motivated by a

simultaneous solution of the TSP together with determination of trajectories between the goals. The approach is based on self-organizing map (SOM) for the TSP that is a two layered neural network originally developed as a visualization technique by T. Kohonen [13]. SOM provides a transformation of a high dimensional input space into a lower dimensional discrete output space, usually a 2D grid. For the TSP, the output layer is a uni-dimensional array of nodes, which represents a path in the input space (called ring) and thus it represents a solution of the TSP, see Fig. 2.

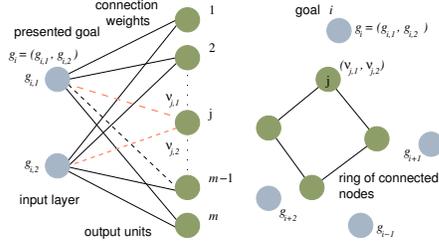


Fig. 2. Structure of the network and projection of the neuron weights into the input space representing the multi-goal path

The unsupervised learning is performed in learning epochs in which all goals are presented to the network in a random order, and neurons compete to become a winner based on its distance to the presented goal. Then, the winner is adapted together with its neighbouring nodes to the presented goal, which can be imagined as a movement of the node to the goal. This procedure is repeated until the network is stabilized and the winners fit the goals. A visualization of the learning is shown in Fig. 3.

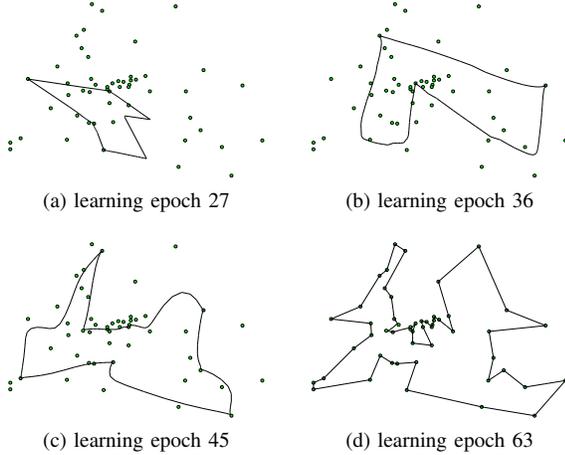


Fig. 3. Visualization of the unsupervised learning of SOM for the TSP

For the multi-goal motion planning problem, we consider SOM adaptation on a graph proposed in [14]. The graph is a robot roadmap and this approach has been successfully used in our previous work [15], where PRM serves as a graph and SOM determines a multi-goal trajectory from which a set of desired areas are covered by an unmanned aerial vehicle considering sensing with a limited range. The main draw back of this solution is that the roadmap has to be created prior the planning and it should be sufficiently dense.

In this paper, we present results of our ongoing effort on simultaneous creation of the motion planning roadmap using the RRG technique and a solution of the underlying TSP on this roadmap; thus, the process of solving the TSP is used to stimulate the roadmap expansion in promising parts of the environment regarding the solution of the TSP. The proposed unsupervised learning exploits the information acquired from the environment and use it for steering the RRG motion planning technique.

The RRG technique builds the roadmap incrementally, therefore it can be used for such an exploitation, whereas PRM technique constructs the roadmap in advance and it is then used for the afterwards path planning queries. Our early achieved results indicate the proposed approach grows the roadmap in desired parts of the environment with respect to the actual goal configuration and the unsupervised learning provides a solution of the multi-goal trajectory problem with increasing quality of solution.

II. PROBLEM STATEMENT

The problem is studied in 3D environment $\mathcal{W} \in \mathbb{R}^3$ represented by a set of triangles forming a set of obstacles \mathcal{O} . Motion planning is realized in configuration space \mathcal{C} of all possible configurations of the robot. The motion planning problem is considered as a problem to find a single trajectory $\kappa : [0, 1] \rightarrow \mathcal{C}_{free}$ from the initial robot configuration $\kappa(0) = q_{init}$ to a goal configuration defined as $d(\kappa(1), q_{goal}) < \varepsilon$, where \mathcal{C}_{free} denotes the obstacle free part of \mathcal{C} as $\mathcal{C}_{free} = \text{cl}(\mathcal{C} \setminus \mathcal{C}_{obst})$, ε is a tolerance for visiting the desired goal location, cl is the closure of a set, and $d(\cdot, \cdot)$ is a distance between two configurations. \mathcal{C}_{obst} represents all configurations for which the robot is in collision with \mathcal{O} .

The motivational scenario in this work is a robotic visual inspection, where the task is to cover n given areas of interest from particular goal locations, which are represented as desired configurations of the robot $\mathcal{G} = \{g_1, \dots, g_n\}$. We assume that each goal configuration is represented by a 6D vector $\mathbb{R}^3 \times SO(3)$ consisted of position (x, y, z) and rotation (α, β, γ) . The robot reaches the goal if it is within the goal in an admissible tolerance ε .

The sequence of visits is defined as (v_1, v_2, \dots, v_n) , where $v_i \in \mathcal{G}$ and $\cup_{1 < i \leq n} v_i = \mathcal{G}$. The multi-goal trajectory in the multi-goal motion planning problem is defined similarly as for the single trajectory as $\tau : [0, 1] \rightarrow \mathcal{C}_{free}$ such that $\tau(0) = \tau(1) = q_{init} \in \mathcal{G}$ and for which there exist $0 \leq t_1 \leq t_2 \leq \dots \leq t_n \leq 1$ and $d(\tau(t_i), v_i) < \varepsilon$. In other words, there exists n configurations of the multi-goal trajectory from which the robot covers requested objects, i.e., they are at least in ε distance from the goals. The MGMP problem can be formulated as follows: *For the given set of goals \mathcal{G} , configuration space \mathcal{C} and admissible distance ε , find trajectory τ^* such that $\tau^* = \arg \min_{\tau \in T} c(\tau)$, where T is the set of all admissible multi-goal trajectories and $c(\cdot)$ is a strictly positive cost function.*

The proposed approach is based on the RRG algorithm [11] used for an incremental construction of the roadmap (graph) $\mathbf{G}_{RRG} = (\mathbf{V}_{RRG}, \mathbf{E}_{RRG})$. The set of vertices \mathbf{V}_{RRG}

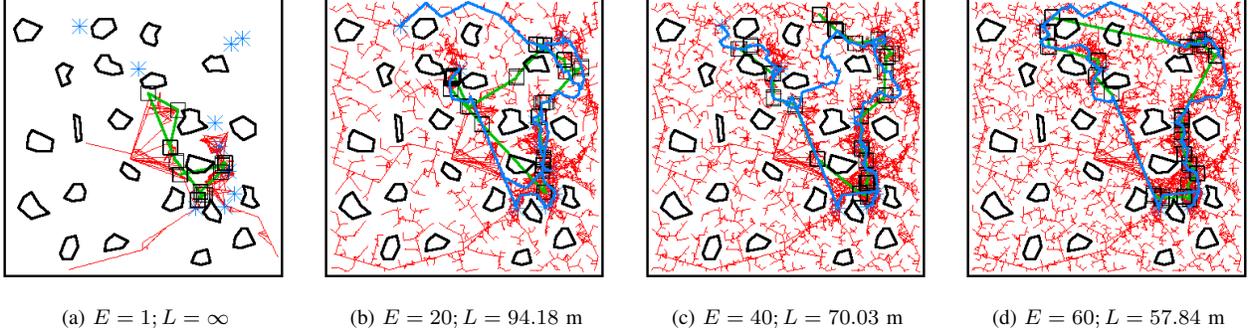


Fig. 4. Examples of solution of the MGMP problem with 10 goals in the *potholes* at the learning epoch E and length of the found multi-goal path L

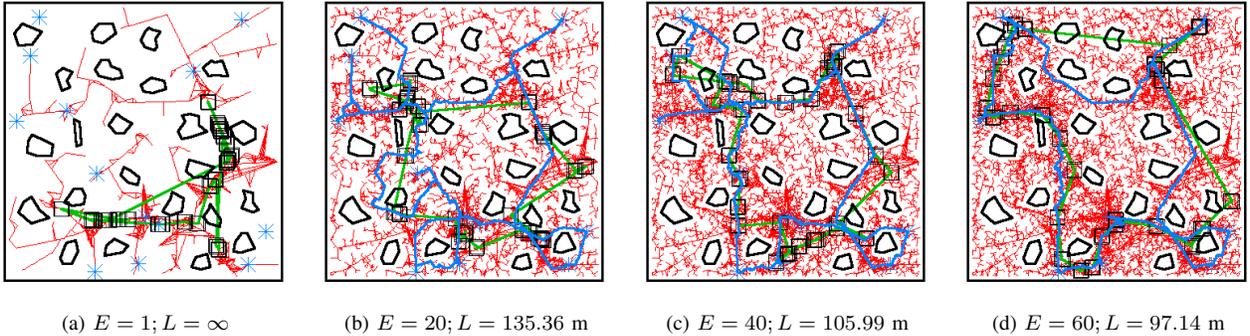


Fig. 5. Examples of solution of the MGMP problem with 13 goals in the *potholes* at the learning epoch E and length of the found multi-goal path L

are configurations of the robot in \mathcal{C}_{free} that are connected by edges $e_i \in \mathbf{E}_{RRG}$, which represent a feasible motion between two configurations. The graph is a result of the roadmap expansion by applying a selected robot control command.

III. PROPOSED MULTI-GOAL TRAJECTORY PLANNING

The output layer of the used neural network forms a ring of m nodes $\mathcal{N} = \nu_1, \dots, \nu_m$ representing a solution of the TSP, where m is selected according to the number of the n goal locations ($m = 2.5n$ nodes are used for all results presented this paper). The adaptation of the network is based on evaluation of the closest neuron to the goal for which we need a distance cost from a node ν to the goal g . In a planar case, we can simply compute the distance using the Euclidean distance, while for the SOM on a graph [14] we can use a length of the shortest paths between the vertices in the graph as nodes and goals are vertices of the graph. However, we do not have such a graph that will connects the nodes (their closest vertices in the graph) with the presented goals, because the graph \mathbf{G}_{RRG} is actually being constructed during solving the TSP. Therefore, we need a suitable approximation of the distance that will use the current knowledge about \mathcal{C}_{free} stored in \mathbf{G}_{RRG} as much as possible.

To maximize such an exploitation of the information stored in the roadmap \mathbf{G}_{RRG} , we propose to consider an intermediate vertex w^* of the graph \mathbf{G}_{RRG} that lies on a trajectory from the node ν to the goal g . Because a complete trajectory from ν to g is not known, such a vertex should

also respect Euclidean approximation of the distance to the goal. We propose to determine such a vertex w with respect to ν and g as

$$w^* = \arg \min_{w \in \mathbf{V}_{RRG}} \left(c(\kappa_{\nu, w}) + |(w, g)|^2 \right) \quad (1)$$

where $c(\kappa_{\nu, w})$ is the trajectory cost from ν to the intermediate node w determined using the current graph \mathbf{G}_{RRG} and $|(w, g)|$ is the Euclidean distance from w to g . The estimated path $P(\nu, g)$ from ν to g is then composed from the trajectory κ_{ν, w^*} in the graph \mathbf{G}_{RRG} and the straight line segment from w^* to g as

$$P(\nu, g) = \kappa_{\nu, w^*} \oplus (w^*, g). \quad (2)$$

This proposed estimation $P(\nu, g)$ is used for determining the winner neuron in the unsupervised learning, and the length of the estimated path is computed as

$$|P(\nu, g)| = c(\kappa_{\nu, w^*}) + |(w^*, g)|, \quad (3)$$

where the cost of the trajectory $c(\cdot)$ is extended by the length of the straight line segment (w^*, g) .

In a single learning epoch, the unsupervised learning of SOM for the TSP sequentially propagates a random permutation of the goals $\Pi(\mathcal{G})$ to the network and the presented goal is used to select the winner node ν^* from the ring of nodes. Then, ν^* is adapted with its neighbors in the direction towards the goal g . The power of the adaptation of the winner and its neighbors is controlled by the fractional learning rate μ and neighboring function $\exp(-\frac{l^2}{\sigma^2})$, where l is a distance of the node from the winner (in the number of nodes) and

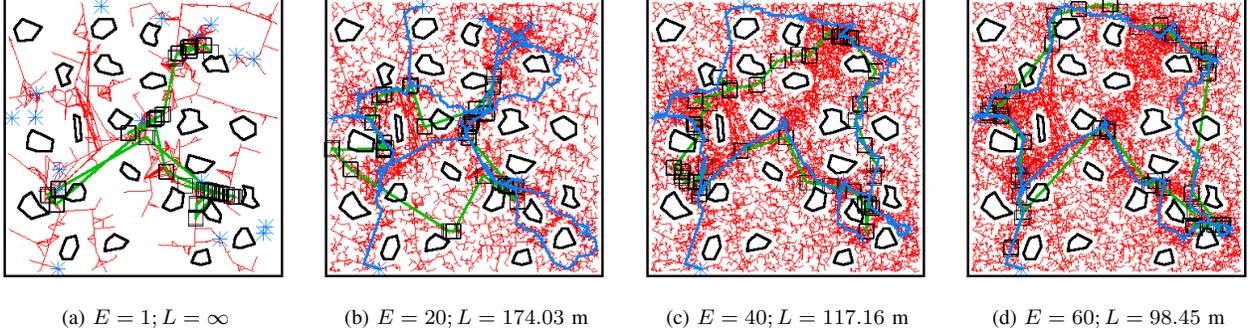


Fig. 6. Examples of solution of the MGMP problem with 17 goals in the *potholes* at the learning epoch E and length of the found multi-goal path L

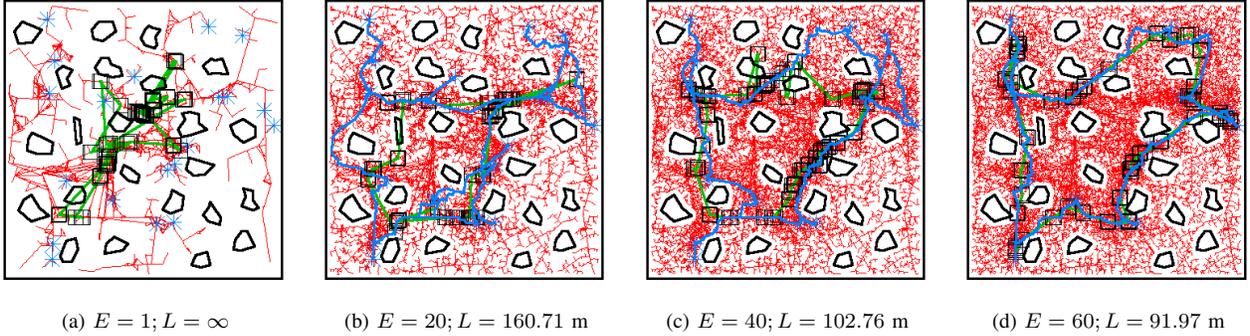


Fig. 7. Examples of solution of the MGMP problem with 20 goals in the *potholes* at the learning epoch E and length of the found multi-goal path L

σ is the learning gain decreased after each learning epoch. The expected traveled distance of the neuron ν towards g is

$$|P(\nu, g)| - |P(\nu', g)| = |P(\nu, g)| \mu e^{-\frac{l^2}{\sigma^2}}, \quad (4)$$

where ν' is a new expected location of the neuron ν . Notice, that such a location of the neuron ν' can be out of the current roadmap. This situation is used for steering the expansion of \mathbf{G}_{RRG} towards the direction of $P(\nu, g)$. But afterwards, the location of the neuron ν' is restricted to an existing vertex of \mathbf{G}_{RRG} when the roadmap was expanded. The proposed adaptation to steer the sampling of $\mathcal{C}_{\text{free}}$ in the multi-goal trajectory planning can be summarized as follows:

- Let the presented goal to the network be g and its winner found by (3) be ν^* ;
- Then, for each ν_i from the set defined by ν^* and its neighbouring nodes:
 - 1) Find $w^* \in \mathbf{V}_{\text{RRG}}$ using (1);
 - 2) Determine the expected position ν'_i using (4);
 - 3) Expand the motion roadmap \mathbf{G}_{RRG} towards ν'_i using goal biasing;
 - 4) Find $w^* \in \mathbf{V}_{\text{RRG}}$ using (1) in the expanded \mathbf{G}_{RRG} and determine new expected position ν'_i ;
 - 5) Restrict ν'_i to w^* if ν'_i would be out of the graph \mathbf{G}_{RRG} ;
 - 6) Set the weights of ν_i to ν'_i ;

The important part of the proposed algorithm is the third step, where the roadmap is expanded. Here, the configuration space is explored if the expected node position ν'_i would be

out of the graph; otherwise the roadmap is enhanced.

IV. CASE STUDY

The proposed approach has been studied in 20×20 meters large environment called *potholes* and for the PhantomX hexapod walking robot (see Fig. 1) with an omnidirectional motion capability. We studied behavior in different types of complexity of the underlying TSP with n randomly generated goals $n \in \{5, 7, 10, 13, 17, 20\}$.

Selected results are visualized in Figures 4, 5, 6, and 7. Each sub-figure represents the current state of the solution at the end of the learning epoch E . The cost of the best found multi-goal trajectory is denoted as L , where ∞ represents a situation an admissible solution has not yet been found till the current epoch. The colors in the figures represents: obstacles of the environment (black), the current roadmap (red), the actual best found trajectory (blue), the desired goals to be visited (blue stars), straight line segments connecting the nodes of the SOM ring (green), and SOM nodes (black squares). Notice, the first admissible solution is found in few epochs while in the rest of the learning the solution is improved as the roadmap becomes denser.

Due to stochastic nature of the proposed SOM-based planning approach, 10 trials have been performed for each problem and average values of the studied parameters of the created roadmap are presented in Table I. The column denoted as CPU Time indicates the required computational time using a single core CPU running at 2.6 GHz, which follows the theoretical complexity $O(n^3 \log(n))$ of the unsupervised

learning. The most computationally demanding part of the algorithm is equation (1) that is solved twice per a single adaptation of each winner in one learning epoch.

TABLE I

AVERAGE VALUES OF THE STUDIED ROADMAP PARAMETERS IN E=100

No. of Goals	No. of Vertices	No. of Edges	CPU Time [s]	Path length [m]
5	3137 (85)	24494 (1404)	4.7 (0.3)	51.6 (0.3)
7	4820 (60)	42113 (771)	15.8 (0.6)	42.8 (0.5)
10	6947 (58)	62076 (1546)	59.2 (1.7)	46.8 (1.1)
13	11259 (97)	96305 (1180)	200.4 (4.2)	65.0 (1.7)
17	16326 (69)	146305 (1204)	600.3 (13.8)	69.5 (2.4)
20	19889 (77)	173934 (1576)	1054.1 (74.9)	86.3 (1.4)

V. CONCLUSION

We present a new unsupervised learning strategy for growing motion planning roadmap in the randomized sampling-based RRG motion planner to find a solution of the multi-goal trajectory problem. The proposed technique is based on self-organizing map for the TSP that allows to simultaneously solve the combinatorial part of the MGMP problem together with determination of the trajectories between the goals. Despite of the relatively simple problems considered, the presented results support feasibility of the proposed approach for the multi-goal motion planning. These first results also motivates us for our future work on the proposed unsupervised learning of the configuration space during the motion planning to further improve the multi-goal planning performance by finding a suitable trade-off between exploring the space and exploiting the current roadmap.

REFERENCES

- [1] G. A. Hollinger, U. Mitra, and G. S. Sukhatme, "Autonomous data collection from underwater sensor networks using acoustic communication," in *IROS*, 2011, pp. 3564–3570.
- [2] B. Englot and F. S. Hover, "Three-dimensional coverage planning for an underwater inspection robot," *Int. J. Rob. Res.*, vol. 32, no. 9-10, pp. 1048–1073, 2013.
- [3] M. Saha, G. Sánchez-Ante, and J.-C. Latombe, "Planning multigoal tours for robot arms," in *ICRA*, vol. 3, 2003, pp. 3797–3803.
- [4] E. Kolakowska, S. F. Smith, and M. Kristiansen, "Constraint optimization model of a scheduling problem for a robotic arm in automatic systems," *Robotics and Autonomous Systems*, vol. 62, no. 2, pp. 267–280, 2014.
- [5] N. Michael, E. Stump, and K. Mohta, "Persistent surveillance with a team of mavs," in *IROS*, Sept 2011, pp. 2708–2714.
- [6] D. L. Applegate, R. E. Bixby, V. Chvatal, and W. J. Cook, *The Traveling Salesman Problem: A Computational Study (Princeton Series in Applied Mathematics)*. Princeton University Press, 2007.
- [7] D. Applegate, R. Bixby, V. Chvátal, and W. Cook, "CONCORDE TSP Solver," 2003, [cited 28 Feb 2012]. [Online]. Available: <http://www.tsp.gatech.edu/concorde.html>
- [8] J. H. Reif, "Complexity of the mover's problem and generalizations," in *Proceedings of the 20th Annual Symposium on Foundations of Computer Science*. IEEE Computer Society, 1979, pp. 421–427.
- [9] L. Kavradi, P. Svestka, J.-C. Latombe, and M. Overmars, "Probabilistic roadmaps for path planning in high-dimensional configuration spaces," *IEEE Transactions on Robotics and Automation*, vol. 12, no. 4, pp. 566–580, 1996.
- [10] S. M. Lavalle, *Planning Algorithms*. Cambridge University Press, 2006.
- [11] S. Karaman and E. Frazzoli, "Sampling-based algorithms for optimal motion planning," *Int. J. Rob. Res.*, vol. 30, no. 7, pp. 846–894, 2011.
- [12] M. Saha, T. Roughgarden, J.-C. Latombe, and G. Sánchez-Ante, "Planning Tours of Robotic Arms among Partitioned Goals," *Int. J. Rob. Res.*, vol. 25, no. 3, pp. 207–223, 2006.
- [13] T. Kohonen, M. R. Schroeder, and T. S. Huang, Eds., *Self-Organizing Maps*, 3rd ed. Springer-Verlag New York, Inc., 2001.
- [14] T. Yamakawa, K. Horio, and M. Hoshino, "Self-Organizing Map with Input Data Represented as Graph," in *Neural Information Processing*. Springer Berlin / Heidelberg, 2006, pp. 907–914.
- [15] P. Janoušek and J. Faigl, "Speeding up coverage queries in 3d multi-goal path planning," in *ICRA*, 2013, pp. 5067–5072.