# Species interaction in a toy ecosystem

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Phy 581: Nonlinear Science and Mathematical Biology

# Introduction.

- A toy ecosystem with predatory and competitive interactions.
- Attempt to model the interaction with logistic equation and allee effect
- Wami effect and our simple model.

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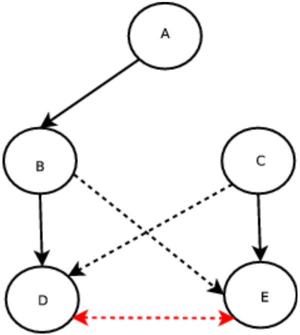
# Outline

# Interacting species

An isolated toy ecosystem with species interactions[1] :

1. Direct Food Chain A⟶B⟶D
2. Exploitative Competition B⟶D⟵C
3. Apparent Competition D⟵B⟶E
4. Indirect Mutualism D⟷E

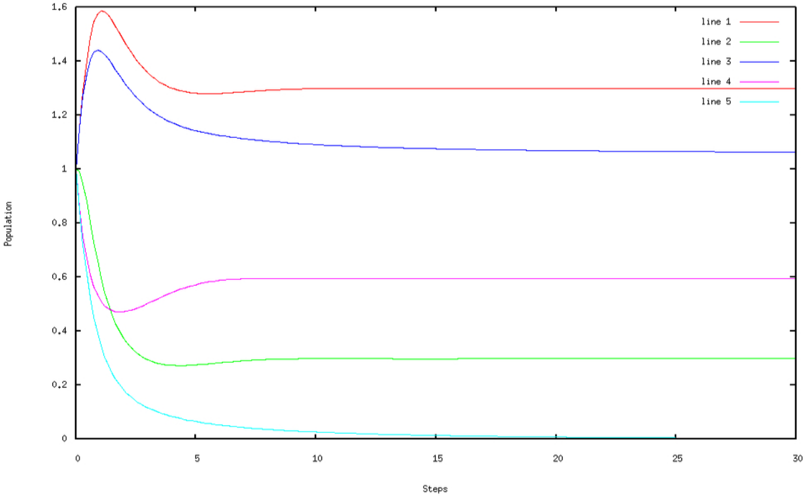


# The equations

$$
\begin{aligned}
\frac{dn_A}{dt} &= a_1 n_A - b_1 {n_A}^2 + c_1 n_A n_B \\
\frac{dn_B}{dt} &= a_2 n_B - b_2 {n_B}^2 - c_1 n_A n_B + d_2(1-r) n_B n_D + d_2 r n_B n_E \\
\frac{dn_C}{dt} &= a_3 n_C - b_3 {n_C}^2 + c_3(1-r) n_C n_E + c_3 r n_C n_D \\
\frac{dn_D}{dt} &= a_4 n_D - b_4 {n_D}^2 - d_2(1-r)) n_B n_D - c_3 r n_C n_D \\
\frac{dn_E}{dt} &= a_5 n_E - b_5 {n_E}^2 - d_2 r n_B n_E - c_3(1-r) n_C n_E
\end{aligned}
$$

# Fixed Points

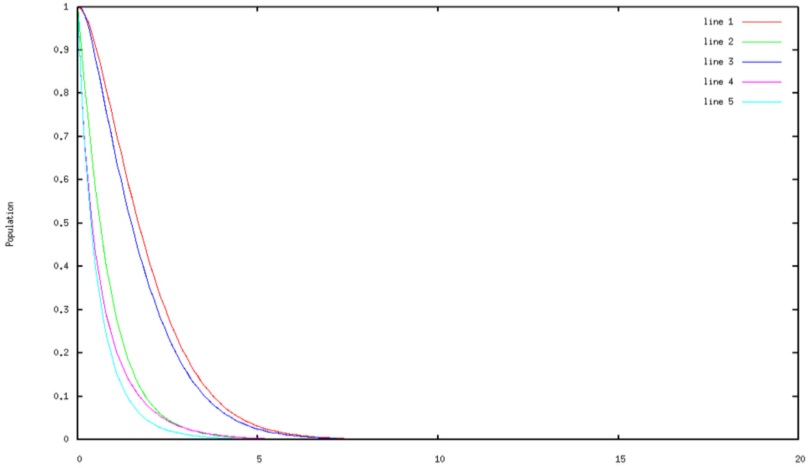
$$
\begin{aligned}
n_A^* &= 0, \frac{a_1 + c_1 n_B}{b_1} \\
n_B^* &= 0, \frac{a_2 - c_1 n_A + d_2(n_D + rN_E)}{b_2} \\
n_C^* &= 0, \frac{a_3 + c_3(n_E + rN_D)}{b_3} \\
n_D^* &= 0, \frac{a_4 - d_2 n_B - c_3 rN_C}{b_4} \\
n_E^* &= 0, \frac{a_5 - d_2 rn_B - c_3 N_C}{b_5}
\end{aligned}
$$


1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Population
0
5
10
15
20
25
30
Steps
line 1
line 2
line 3
line 4
line 5

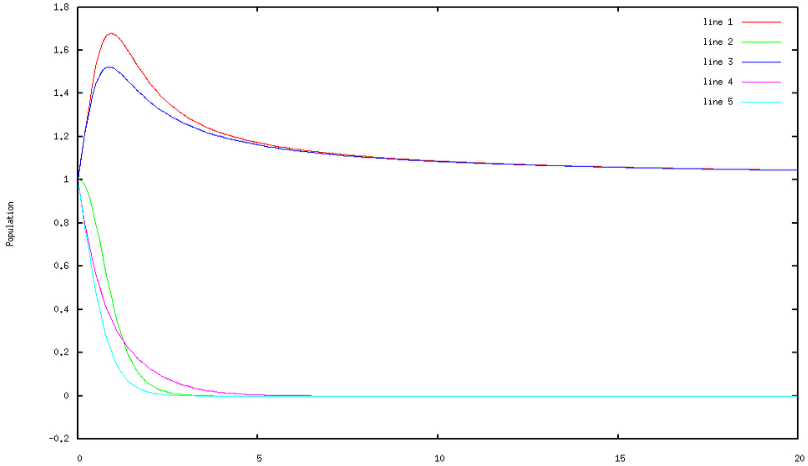
# The *cubic* effect

$$
\begin{aligned}
\frac{dn_A}{dt} &= a_1 {n_A}^2 - b_1 {n_A}^3 - l_1 n_A + c_1 n_A n_B \\
\frac{dn_B}{dt} &= a_2 {n_B}^2 - b_2 {n_B}^3 - l_2 n_B - c_1 n_A n_B \\
&\quad + d_2(1-r) n_B n_D + d_2 r n_B n_E \\
\frac{dn_C}{dt} &= a_3 {n_C}^2 - b_3 {n_C}^3 - l_3 n_C - c_3(1-r) n_C n_E \\
&\quad + c_3 r n_C n_D \\
\frac{dn_D}{dt} &= a_4 {n_D}^2 - b_4 {n_D}^3 - l_4 n_D - d_2(1-r) n_B n_D \\
&\quad - c_3 r n_C n_D \\
\frac{dn_E}{dt} &= a_5 {n_E}^2 - b_5 {n_E}^3 - l_5 n_E - d_2 r n_B n_E - c_3(1-r) n_C n_E
\end{aligned}
$$

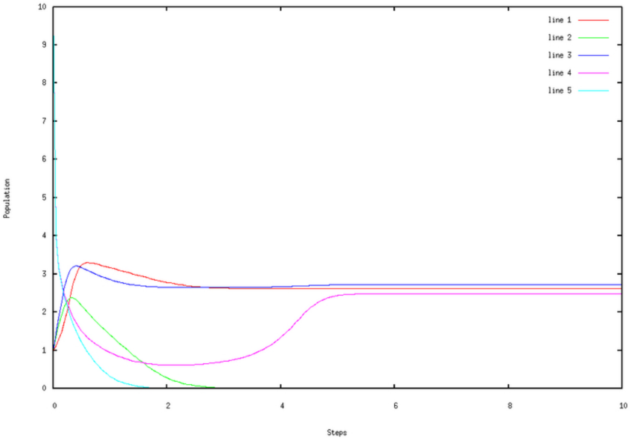
# The cubic equation for all $a_i = 1$ for all i



# The cubic equation for all $a_i = 2$ for all i



# D and E with initial population of 10.

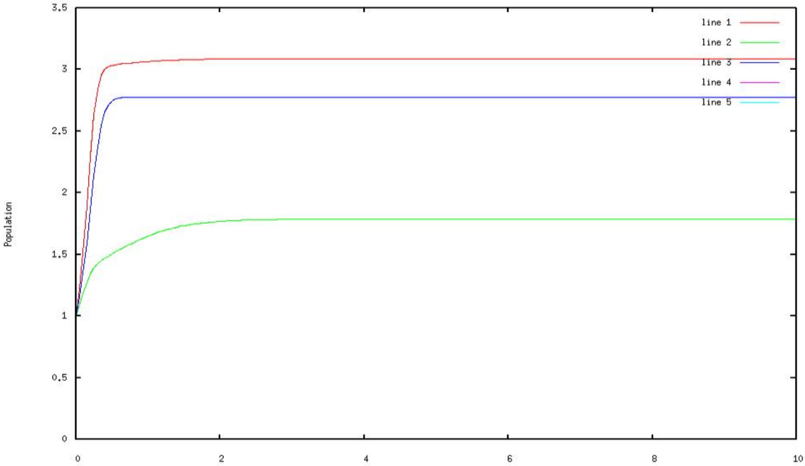


# The *Allee* effect: max growth near mid population.

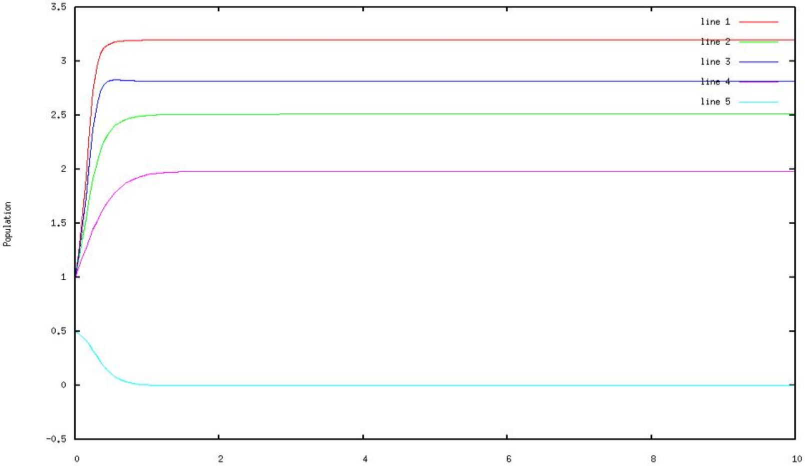
$$\dot{N}/N = r - a(N-b)^2$$

$$\begin{aligned}
\frac{dn_A}{dt} &= 2a_1 b_1 {n_A}^2 - b_1 {n_A}^3 + (l_1 - a_1^2) n_A + c_1 n_A n_B \\
\frac{dn_B}{dt} &= 2a_2 b_2 {n_B}^2 - b_2 {n_B}^3 + (l_2 - a_2^2) n_B - c_1 n_A n_B \\
&\quad + d_2 (1-r) n_B n_D + d_2 r n_B n_E \\
\frac{dn_C}{dt} &= 2a_3 b_3 {n_C}^2 - b_3 {n_C}^3 + (l_3 - a_3^2) n_C - c_3 (1-r) n_C n_E \\
&\quad + c_3 r n_C n_D \\
\frac{dn_D}{dt} &= 2a_4 b_4 {n_D}^2 - b_4 {n_D}^3 + (l_4 - a_4^2) n_D - d_2 (1-r) n_B n_D \\
&\quad - c_3 r n_C n_D \\
\frac{dn_E}{dt} &= 2a_5 b_5 {n_E}^2 - b_5 {n_E}^3 + (l_5 - a_5) n_E - d_2 r n_B n_E - c_3 (1-r) n_C n_E
\end{aligned}$$

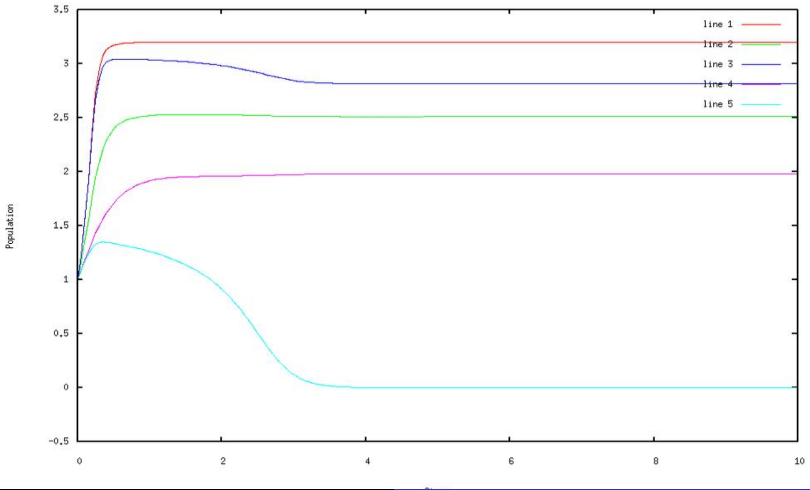
# Allee effect Population without *D* and *E*



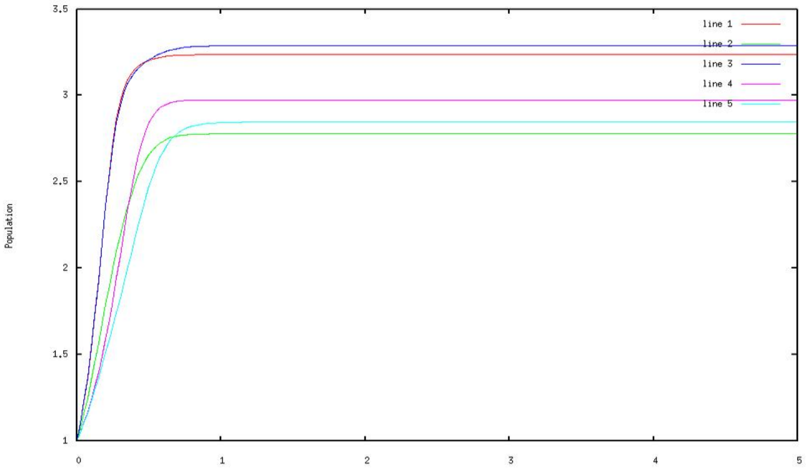
# Allee effect: *E* with population 0.5



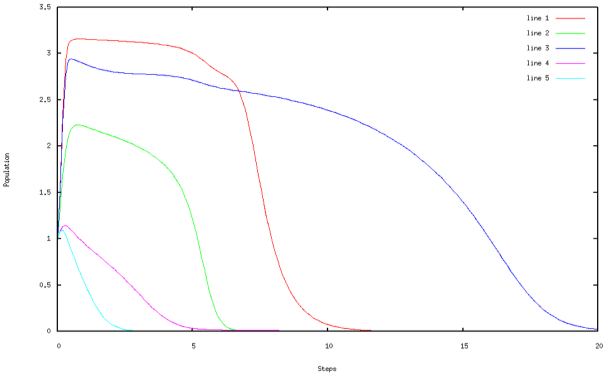
# Allee: $b_i = 2$ for all i, $a_4 = a_5 = 1.5$



# Allee: $b_i = 2$ for all i, $a_4 = a_5 = 2$

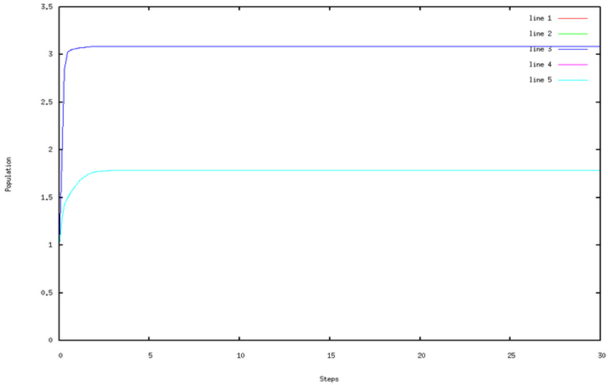


# Wami! effect: Population should disappear without food

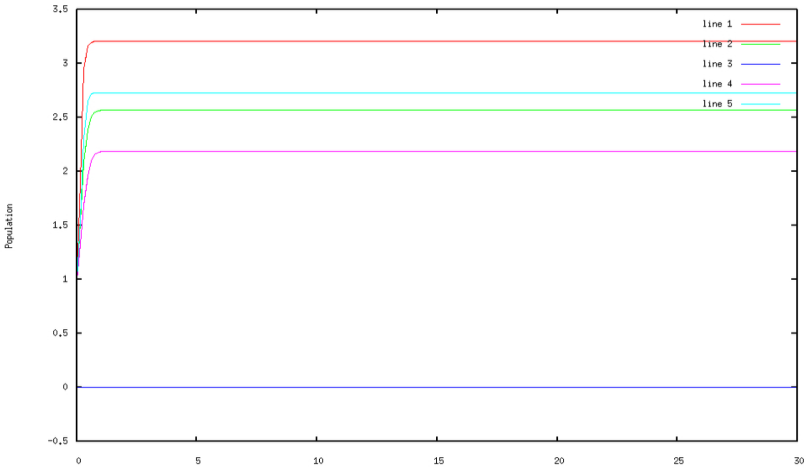


$$\begin{aligned}
\frac{dn_A}{dt} &= (2a_1 b_1 {n_A}^2 - b_1 {n_A}^3)(\frac{n_B}{n_B + 0.01}) + (l_1 - a_1^2)n_A + c_1 n_A n_B \\
\frac{dn_B}{dt} &= (2a_2 b_2 {n_B}^2 - b_2 {n_B}^3)(\frac{n_D + n_E}{n_D + n_E + 0.01}) + (l_2 - a_2^2)n_B \\
&\quad - c_1 n_A n_B + d_2(1 - r_2)n_B n_D + d_2 r_2 n_B n_E \\
\frac{dn_C}{dt} &= (2a_3 b_3 {n_C}^2 - b_3 {n_C}^3)(\frac{n_D + n_E}{n_D + n_E + 0.01}) + (l_3 - a_3^2)n_C \\
&\quad - c_3(1 - r_3)n_C n_E + c_3 r_3 n_C n_D \\
\frac{dn_D}{dt} &= 2a_4 b_4 {n_D}^2 - b_4 {n_D}^3 + (l_4 - a_4^2)n_D - d_2(1 - r_2)n_B n_D - c_3 r n_C n \\
\frac{dn_E}{dt} &= 2a_5 b_5 {n_E}^2 - b_5 {n_E}^3 + (l_5 - a_5)n_E - d_2 r_2 n_B n_E - c_3(1 - r_3)n_C
\end{aligned}$$

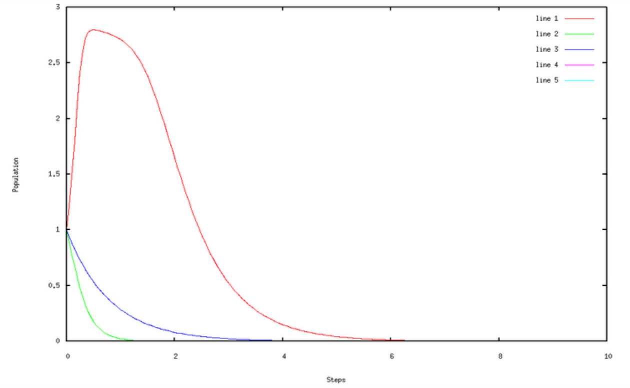
# Population without $A$



# Population without C

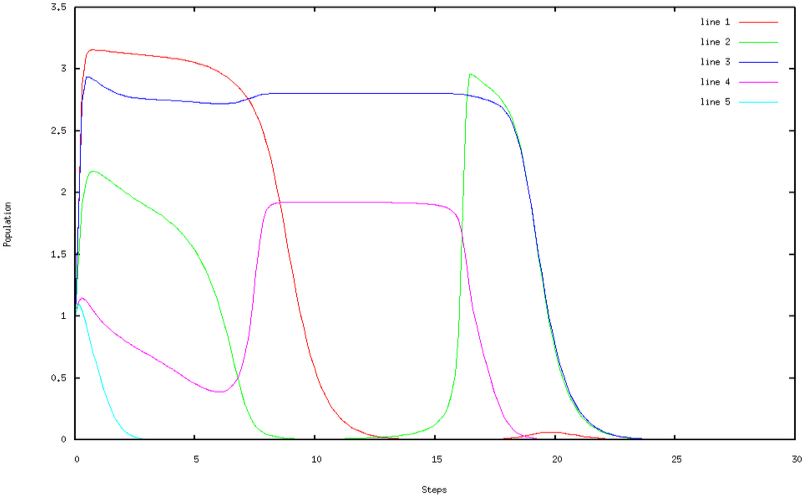


# Population without D and E



# Outline

## An interesting effect



$$\frac{dn_A}{dt} = (2a_1 b_1 {n_A}^2 - b_1 {n_A}^3)(\frac{n_B}{n_B + 0.1}) + (l_1 - a_1^2)n_A + c_1 n_A n_B$$

etc ...

# Future work

- Extend to a more 'open' ecosystem, connecting to meta ecosystems.
- Empirical data to find useful values for the parameters.
- Self-healing properties of ecosystem, on perturbations.

# Thanks. Questions?