

# Angel and Shreiner: Interactive Computer Graphics, Seventh Edition

## Chapter 11 Odd Solutions

11.1  $(m+1)^3$

11.3 As  $u$  varies over  $(a, b)$ ,  $v = \frac{u-a}{b-a}$  varies over  $(0, 1)$ . Substituting into the polynomial  $p(u) = \sum_{k=0}^n c_k u^k$ , we have  $q(v) = \sum_{i=0}^n d_i v^i = \sum_{k=0}^n c_k ((b-a)v + a)^k$ . We can expand the products on the right and match powers of  $v$  to obtain  $\{d_i\}$ .

11.5 Consider the Bernstein polynomial

$$b_{kd}(u) = \binom{d}{k} u^k (1-u)^{d-k}.$$

For  $k=0$  or  $k=d$ , the maximum value of 1 is at one end of the interval  $(0,1)$  and the minimum is at the other because all the zeros are at 1 or 0. For other values of  $k$ , the polynomial is 0 at both ends of the interval and we can differentiate to find that the maximum is at  $u = k/d$ . Substituting into the polynomial, the maximum value is  $\frac{d!}{d^d} \frac{k^k}{k!} \frac{(d-k)^{d-k}}{(d-k)!}$  which is always between 0 and 1.

11.7 Any quadric can be written as

$$q(x, y, z) = ax^2 + by^2 + cz^2 + 2dxy + 2exz + 2fyz + 2gx + 2hy + 2iz + j = 0,$$

where  $a, b, c, d, e, f, g, h, i$  and  $j$  are constants. Let  $\mathbf{p}^T = \begin{bmatrix} x & y & z & 1 \end{bmatrix}$ . Then, we can rewrite the equation as

$$q(\mathbf{p}) = \mathbf{p}^T \mathbf{Q} \mathbf{p} = 0,$$

where

$$\mathbf{Q} = \begin{bmatrix} a & d & e & g \\ d & b & f & h \\ e & f & c & i \\ g & h & i & j \end{bmatrix}.$$

Note that we can also use  $\mathbf{p}^T = \begin{bmatrix} x & y & z & w \end{bmatrix}$  where  $w$  can be any constant.

11.15 For  $r=0$  we get the line between  $P_0$  and  $P_2$ . For  $r = \frac{1}{2}$  we get the parabola  $u^2 P_0 + 2u(1-u)P_1 + (1-u)^2 P_2$  which passes through  $P_0$  and  $P_2$ .

For  $r > \frac{1}{2}$ , we obtain hyperbolas, and for  $r < \frac{1}{2}$ , we obtain ellipses. Thus, we can use NURBSs to obtain both parametric polynomial curves and surfaces, and to obtain quadric surfaces.

11.17 We can write the Hermite surface as

$$\mathbf{p}(u, v) = \mathbf{u}^T \mathbf{M}_H \mathbf{Q} \mathbf{M}_H^T \mathbf{v} = \mathbf{u}^T \mathbf{A} \mathbf{v},$$

where  $\mathbf{Q}$  contains the control point data and  $\mathbf{M}_H$  is the Hermite geometry matrix. If evaluate  $\mathbf{p}$ ,  $\frac{\partial \mathbf{p}}{\partial u}$ ,  $\frac{\partial \mathbf{p}}{\partial v}$ , and  $\frac{\partial^2 \mathbf{p}}{\partial u \partial v}$  at the corners we find that the 16 values in the matrix  $\mathbf{A}$  are the 4 values at the 4 corners of the patch, the first partial derivatives  $\frac{\partial \mathbf{p}}{\partial v}$  and  $\frac{\partial \mathbf{p}}{\partial u}$  at the corners and the first mixed partial derivative  $\frac{\partial^2 \mathbf{p}}{\partial u \partial v}$  at the corners

11.19 This process creates a quadric curve which interpolates  $P_0$  and  $P_2$  and lies in the triangle defined by  $P_0$ ,  $P_1$ , and  $P_2$

11.21 Nothing unusual happens other than the slope at  $u = 0$  must be zero as long as the control points are still separated in parameter space.

11.25 The columns of the matrix  $\mathbf{M}_R$  contain the coefficients of the blending polynomials which are

$$\begin{aligned} p_0(u) &= -u^3 + 2u^2 - u, \\ p_1(u) &= 2u^3 - 5u^2 + 2, \\ p_2(u) &= -3u^3 + 4u^2 - u, \\ &u^3 - u^2. \end{aligned}$$

Note that the third and fourth polynomials can be obtained from the first and second by substituting  $1 - u$  for  $u$ . We zeros of the fourth polynomial are 0, 0, and 1 so the zeros of the first are 0, 1, and 1. We can obtain the zeros of the third by factoring out  $u$  which gives a zero at 0 and solving the resulting quadratic equation to find the zeros at  $\frac{-3 \pm \sqrt{7}}{2}$ . The zeros of the second polynomial are thus 1 and  $\frac{1 \pm \sqrt{7}}{2}$ .