Homework set 4: Grammars, regular languages, and context-free languages — due Monday 5 February

Total number of points available on this homework is 150. Full credit is equivalent to 100 points.

1. (20 pts.) What is the language generated by the grammar $S \to SSS \mid a$?
2. (20 pts.) Write a grammar for the language of the regular expression $((a + b^\star c)d)^\star$.
3. (30 pts.) Give examples of languages that satisfy the following conditions:
   (a) L_1 is regular and infinite, L_2 is not regular, and $L_1 \subset L_2$.
   (b) L_3 is regular, L_4 is not regular, and $L_4 \subset L_3$.
4. (30 pts.) The reverse of a string x is denoted x^R, and is defined recursively as follows: $\varepsilon^R = \varepsilon$, and for $\sigma \in \Sigma$, $(x\sigma)^R = \sigma x^R$.

   Consider the following grammar G over $\Sigma = \{a, b\}$:

   $$S \to aSa$$
   $$S \to bSb$$
   $$S \to \varepsilon$$

   Prove that it is possible to derive from S any string of the form xSx^R, where x is any string over $\{a, b\}$.
5. (50 pts. - extra credit) Show that English is not context-free.

If you wish to study formal languages beyond the brief introduction we were able to do in class, here are some good books, ordered from more theoretical to more applied:

- Harry Lewis and Christos Papadimitriou: *Elements of the Theory of Computation*, Prentice-Hall, 1981
- György E. Révész: *Introduction to Formal Languages*, Dover, 1991, $6.95
- Alfred V. Aho and Jeffrey D. Ullman: *The Theory of Parsing, Translation, and Compiling*, (2 vols.), Prentice-Hall, 1972-73
- Alfred V. Aho, Ravi Sethi, and Jeffrey D. Ullman: *Compilers: Principles, Techniques, and Tools*, Addison-Wesley, 1988