Homework set 6: Programming in Prolog — due Wednesday 21 February

Total number of points available on this homework is 100. Full credit is equivalent to 100 points.

1. (100 pts.) Recall from class the language over the alphabet $\{a,b,c\}$, $L = \{a^n b^n c^n \mid n \in \mathbb{N}\}$. It is generated by the following grammar:

$$G = (\{S,X,Y\},\{a,b,c\},S,F),$$

where the productions are:

$$F = \{S \to abc, S \to aXbc, Xb \to bX, Xc \to Ybcc, bY \to Yb, aY \to aaX, aY \to aa\}$$

This grammar is not context-free, because on the left-hand sides of some productions there are strings longer than single non-terminals. (In fact, there is no context-free grammar for this language.) However, the productions in this grammar satisfy a *length-increasing* property: the number of symbols on the right is never less than the number on the left. Such grammars are called context-sensitive. (Note that there are grammars, and their languages, that are not even context-sensitive.)

Write a Prolog program that determines whether a string belongs to the language L. Strings over the alphabet $\{a,b,c\}$ will be represented as Prolog lists of Prolog atoms `a`, `b`, and `c`. You should split the program into a set of predicates that are general-purpose (for any context-sensitive grammar), and a set of predicates that describe the grammar G.

The program should result in queries like these:

```
?- accepts([]).
No
?- accepts([a]).
No
?- accepts([a,b]).
No
?- accepts([a,b,b]).
No
?- accepts([a,b,c]).
Yes
?- accepts([a,a,b,b,c,c]).
Yes
?- accepts([a,a,a,a,a,a,b,b,b,b,b,b,c,c,c,c,c,c]).
Yes
?- accepts([a,a,a,a,a,a,b,b,b,b,b,b,c,c,c,c,c,c,a]).
No
```

You may find useful the predefined predicate `length/2`, which works as follows: given a list `X`, the goal `length(X,N)` succeeds and sets N to the integer equal to the length of the list `X`.

You should test your program on the queries above and others you find useful to cover the various cases of strings in the language and outside the language. Make sure that your program does not go into an infinite loop on any inputs. Do not submit your program unless it has passed all these tests.

Submit the listing of the program and your test queries, as hard copy.