Homework set 8: λ-calculus — due Monday 26 February

Total number of points available on this homework is 100. Full credit is equivalent to 100 points.

1. (10 pts.) Identify the bound and free variables of each of the following lambda expressions:

   (a) $(\lambda x.xy)(\lambda y.y)$

   (b) $(\lambda x.\lambda y.xz(yz))\,(\lambda x.y(\lambda y.y))$

2. (30 pts.) For each of the following λ-expressions (with predefined arithmetic):

   (a) $\lambda x.\lambda y.(\lambda z.z)x(+\,y\,1)$

   (b) $(\lambda h.(\lambda x.h(xx))(\lambda x.h(xx)))\,((\lambda x.x)(+\,1\,5))$

   do the following:

   - underline all of the redexes indicating whether they are β, η, or δ redexes;
   - identify the leftmost-outermost and leftmost-innermost redexes;
   - write down the normal form of each, showing the reduction steps required to reach them, assuming normal-order reduction.

3. (20 pts.) The three fundamental combinators are:

   $\mathbf{I} \triangleq \lambda x.x$
   $\mathbf{K} \triangleq \lambda xy.x$
   $\mathbf{S} \triangleq \lambda xyz.xz(yz)$

   Show that $\mathbf{SKK} = \mathbf{I}$.

4. (40 pts.) Find the normal form of:

   (a) $\mathbf{K}(\mathbf{SII})$

   (b) $\mathbf{S}(\mathbf{S}(\mathbf{KS})(\mathbf{KI}))(\mathbf{KI})$