

CS 201 Mathematical Foundations of Computer Science

Test #1

This test is in-class, open-book, open-notes; total time is 50 minutes. There are 4 questions, each worth the same number of points.

Problem 1: Negate the following two logical expressions:

$$\forall x \exists y [(\exists z [z \leftrightarrow \neg y]) \wedge (x \rightarrow y)]$$

$$\exists x \exists y \forall z [(x \rightarrow (y \wedge z)) \vee (x \wedge z)]$$

Problem 2: Prove by direct construction: a number n is divisible by 19 if and only if (note the two directions!) the number of tens plus twice the number of units is divisible by 19. That is, if we write $n = 10x + y$, then we are saying that n is divisible by 19 if and only if $x + 2y$ is divisible by 19.

Problem 3: Prove by induction:

$$\sum_{i=1}^n i \cdot (i + 1) = \frac{n \cdot (n + 1) \cdot (2n + 4)}{6}$$

Problem 4: Consider the following small piece of code

```
z <- x
w <- 0
while w < y do
  z <- z*z
  w++
return z
```

Verify that it computes $z = x^{2^y}$ by asserting and proving the loop invariant $z = x^{2^w}$.

Be as formal as possible in your write-up. For each theorem to prove, state the hypothesis, the conclusion, which proof method you are using. When using induction, state what needs to be proved in the basis; state what is the inductive hypothesis and what needs to be proved in the step.