CS 261, HW1

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Due: Jan 29th

1. Let p and q be the propositions "You are an engineer" and "You love plaid" respectively. Express each of the following compound propositions as an English sentence.

   (a) $\neg p$

   (b) $\neg p \wedge q$

   (c) $\neg q \Rightarrow \neg p$

   (d) $\neg q \vee (\neg p \wedge q)$

2. Let p, q and r be the following propositions: "You are a mathematician"; "You are an engineer"; and "You are a sharp dresser" respectively. Write these propositions using p, q and r and logical connectives.

   (a) You are a mathematician but you are not an engineer

   (b) You are a mathematician, you are an engineer and you are a sharp dresser

   (c) If you are a sharp dresser, you must be a mathematician

   (d) Being a mathematician and an engineer is sufficient to be a sharp dresser

   (e) You are a sharp dresser if and only if you are a mathematician or you are an engineer

3. Excercise 1-17-24: State the converse, contrapositive and inverse of each of these conditional statements

   (a) If it snows tonight, then I will stay home

   (b) I go to the beach whenever it is a sunny day

   (c) When I stay up late, it is necessary that I sleep until noon

4. You are on an island where all people are either *truth tellers*, who always tell the truth, or *liars*, who never tell the truth. A person on this island is accused of a crime, and hires an attorney. The defendant is publicly known to be a truth teller. The following exchange takes place in court:
Attorney: "If the defendant committed the crime, he had an accomplice."
Defendant: "That is not true!"
Did the attorney help his client? Justify your answer.

5. What if the Attorney was publicly known to be a liar and she says "If the defendant committed the crime, he did *not* have an accomplice." Does the attorney help or hurt his client, or neither? Justify your answer. Hint: Let p be the proposition "The defendant committed the crime" and q be the proposition "The defendant had an accomplice". Use De Morgan's rule!

6. Show that $p \Rightarrow q$ and $\neg q \Rightarrow \neg p$ are logically equivalent using either truth tables or rules of logical equivalence (Table 6 and 7)

7. Show that $((p \vee q) \wedge (\neg p \vee r)) \Rightarrow (q \vee r)$ is a tautology.

8. How many of the following disjunctions can be made simultaneously true by an assignment of truth values to p, q and r: $p \vee q$, $\neg p \vee r$, $\neg p \vee \neg r$, $\neg p \vee \neg q$, $\neg r \vee p$.

9. Exercise 1.3.6

10. Exercise 1.3.38

11. Exercise 1.3.42

12. You are lounging on the beach on the island of liars and truth tellers with a large group of natives and you hear the following exchange:
Alice: "We are all liars and Bob is a truth teller"'
Bob: "We are all liars or Carol is a liar"
What can you say about Alice, Bob and Carol? Justify your answer. Hint: Let $L(x)$ be the proposition that x is a liar; use quantifiers and De Morgan's laws for quantifiers in your answer. Consider the two cases where Alice is a truth teller or Alice is a liar.

13. *Challenge*: You are investigating a murder on the island of liars and truth tellers. You have assembled a group and you want to know if

the murderer is in that group. You know that the murderer is a liar. However, you don't know which members of the group, if any are truth tellers. Moreover, you are only allowed to ask yes/no questions to the leader of the group and you want to minimize the number of questions you ask. Assume that everyone has perfect information about who is the murderer, and the fact that he is a liar. Hint: Your questions may use propositional logic and quantifiers. Let $M(x)$ be the proposition, "person x is a murderer in the group". Let $L(x)$ be the proposition "person x is a liar".

- Show how to determine if the murderer is in the group by asking two yes/no questions to the group leader.

- Show how to determine if the murderer is in the group by asking a single yes or no question to the leader. Hint: For a person x, let $Say - Murderer(x)$ be a proposition that is true if x would say that there is a murderer in the group.