

CS 561, HW 2

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Remember: you are encouraged to work on the homework in groups, but please observe the “Star Trek” rule from the syllabus.

1. Problem 7-3 (Alternative quicksort analysis)
2. You are doing a stress test on a particular model of smart phones. You have a ladder with n rungs. You want to determine the highest rung from which you can drop a phone without it breaking and you want to do it with the smallest number of phone drops.
 - (a) Imagine that you have exactly 2 phones. Devise an algorithm that can determine the highest safe rung using $o(n)$ drops.
 - (b) Now suppose you have k phones. Devise an algorithm that can determine the highest safe rung with the smallest number of drops. If $f_k(n)$ is the number of drops that your algorithm needs, what is $f_k(n)$ asymptotically? Hint: you should ensure that $f_k(n) = o(f_{k-1}(n))$ for any $k \geq 2$.
3. The game of *Match* is played with a special deck of 27 cards. Each card has three attributes: color, shape and number. The possible color values are {red, blue, green}, the possible shape values are {square, circle, heart}, and the possible number values are {1, 2, 3}. Each of the $3 * 3 * 3 = 27$ possible combinations is represented by a card in the deck. A match is a set of 3 cards with the property that for every one of the three attributes, either all the cards have the same value for that attribute or they all have different values for that attribute. For example, the following three cards are a match: (3, red, square), (2, blue, square), (1, green, square).
 - If we shuffle the deck and turn over three cards, what is the probability that they form a match? Hint: given the first two cards, what is the probability that the third forms a match?

- If we shuffle the deck and turn over n cards where $n \leq 27$, what is the expected number of matches, where we count each match separately even if they overlap? Note: The cards in a match do not need to be adjacent! Is your expression correct for $n = 27$?
4. Imagine n points are distributed uniformly at random on the perimeter of a circle that has circumference 1. Show that the expected number of pairs of points that are within distance $\Theta(1/n^2)$ of each other is greater than 1. FYI: this problem has applications in efficient routing in peer-to-peer networks.

Hint: Partition the circle into n^2/k regions of size k/n^2 for some constant k ; then use the Birthday paradox to solve for the necessary k .