CS 362, HW 6

Prof. Jared Saia, University of New Mexico

1. Find the optimal parenthesization for a matrix-chain product whose sequence of dimensions is: $(4, 2, 5, 1, 3)$. Include the tables used to compute your result.

2. A thief repeatedly robs the same bank. To avoid capture, he decides to never rob the bank fewer than 10 days after the last robbery. He has obtained information, for the next n days, on the amount of money b_i that is held at the bank on day i.

   (a) Let $r(i)$ be the maximum amount of revenue the thief can safely obtain from day 1 through day i. Give a recurrence relation for $r(i)$.

   (b) Describe a dynamic program based on this recurrence. What is the runtime of your algorithm?

3. Now consider the above problem when: (1) for each day, there is an integer value giving the amount of work w_i that the thief must perform to rob the bank on that day (due to the amount of security on that day); and (2) there is an additional constraint that the sum of work the thief ever performs is less than some value W. Let $r(i, j)$ be the maximum amount of revenue obtainable on days 1 through i, with at most j total work.

   (a) Give a recurrence relation for $r(i, j)$.

   (b) Describe a dynamic program based on this recurrence. What is the runtime of your algorithm?

4. **Greed and Chocolate** You have a chocolate bar consisting of n chunks. Each chunk $1 \leq i \leq n$ has some positive value v_i (for example, chunks with high nougat content are more valuable!)

   You must break the bar into exactly k *parts* to share with your friends, where each part consists of some number of contiguous, unbroken chunks. The value of a part is the sum of the value of all chunks in that part. Your (greedy) friends chose their parts first, and you get the part remaining, i.e the part of smallest value. Thus, your goal is to break in such a way that you maximize the value of the minimum value part.

Give a dynamic program to solve this problem. What is the runtime of your algorithm?