CS 506, HW3

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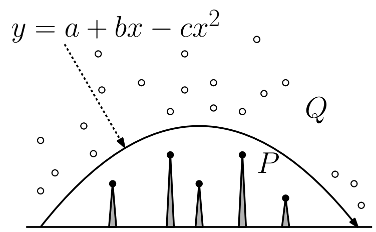

Figure 1. LP

1. (Adapted from Mount F'16) There is a set of n building tops, represented by points $P = \{p_1, \ldots p_n\}$ and m floating balloons, represented by points $Q = \{q_1, \ldots q_m\}$ (Figure 1). You have a cannon in $\mathbb{R}^2$ that has three controls labeled "a", "b", and "c". A projectile shot from this cannon travels along the arc $y = a + bx - cx^2$. Can you adjust the cannon so that the projectile travels above the set P, but below the set Q? You should determine this in $O(n + m)$ time. You can assume anything about the initial location of the canon so long as you clearly state it. Hint: Use Seidel's LP algorithm.
   Next, adapt your algorithm to find, in linear time in $n + m$, if there is a x-degree polynomial that separates any set of points P and Q in $\mathbb{R}^d$ for d and x both $O(1)$. FYI: This has applications to learning polynomial classifiers that separate P and Q.

2. You are given a set of points P in the plane. You want to find the smallest circle that contains all points. Give an efficient algorithm to do this. Hint: This can be done using an incremental algorithm and backwards analysis, as used in Seidel's LP algorithm.

3. Prove that the polar transformation discussed in class is incidence preserving and inclusion reversing.

4. Show that the Johnson-Lindenstrauss projection approximately preserves dot products. In particular, let P be the random projection matrix, let x and y be two unit vectors in the high dimensional space, and then bound the probability that $|x \cdot y - Px \cdot Py| \leq \epsilon$. Hint: Consider $1/4(|P(x+y)|^2 - |P(x-y)|^2)$, and then use what we proved in class about preservation of the norms of difference vectors.

5. Give an example, i.e., a set of n points in $\mathcal{R}^n$, that shows that the Johnson-Lindenstrauss projection does not preserve L_1 distances to within even a factor of 2.