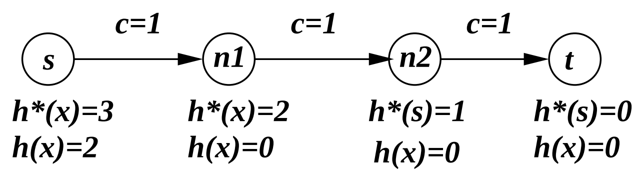
Solution Set to Homework 1

1. **4.6** *Admissibility does* not *imply monotonicity, though monotonicity* does *imply admissibility. A trivial example of a heuristic which is admissible but not monotonic is:*



This heuristic function is admissible, but not monotonic because $h(s) - h(n_1) > c(s, n_1)$. *Effectively, admissibility requires that the heuristic be* globally *consistent – that it underestimate the total path length to the goal. Monotonicity, on the other hand, requires that the heuristic be* locally *consistent – that it underestimate the distance to* every *neighbor of a node (and, inductively, to every node in the graph).*

4.7 *To answer this question in precise detail is actually somewhat complex and cumbersome, but the following answer would suffice for this assignment:*

To show that the set of states opened by A is a subset of that opened by breadth-first search (BFS), it suffices to show that A* doesn't open any nodes that BFS wouldn't open (i.e., if A* opens it, then BFS must as well).*

Proof: (By contradiction) Let $C*$ *be the true cost (distance) from the start state to the goal state. Assume that there exists some node* x' *that BFS doesn't open, but that A* does. We know that BFS opens nodes in order by their distance from the start state, i.e., in order by* $g()$. *Thus, if BFS has not opened* x', *we know that* $g(x') > C*$. *We also know that A* opens nodes in order according to* $f(x) = g(x) + h(x)$, *so if A** has *opened* x', *then* $f(x') < C*$. *Since A* uses the same* $g()$ *that BFS does, this implies that* $h(x) < 0$. *But that conflicts with our definition of* $h(x) \geq 0$ *for all* x. *Thus, there cannot be a node* x' *opened by A* but not by BFS. Therefore, the set of nodes opened by A* is a subset of that opened by BFS.*

A number of people offered arguments of the form "BFS is just A with* $h(x) = 0$ *for all* x. *Thus, A* with any other heuristic is more informed, and a more informed heuristic causes A* to open fewer nodes, therefore, A* must open fewer nodes than BFS." There are two problems with this: first, we never* proved *in class that informedness causes fewer nodes to be opened – I told you that to give you some intuition about what makes a good heuristic. But that's a* consequence *of informedness, not the* definition *of it. This question was, effectively, asking you to prove that this holds. You can't use the conclusion to prove the conclusion. Second, even if you know that A* opens* fewer *nodes than BFS, you don't necessarily know that those nodes are a* subset. *It* might *be the case that A* opens a very small set of nodes, but goes by a very different and roundabout path, examining some things that BFS never would, but still somehow finding the goal every time. Only by showing that A* never examines something that BFS wouldn't touch, can we know for sure that this doesn't happen.*

2. Prepsi Cola Inc. produces two different beverages: their flagship product, Prepsi Cola, and their high-caffeine swill for hacker types and extreme athlete wanna-be's, Mountain Drool. They have two production plants, one in Kentucky (where labor is cheap, but shipping to large markets is expensive) and one in Massachusetts (where labor is expensive but shipping is cheap). Let the cost of producing a unit of Prepsi Cola in Kentucky be $C_{p,k}$, the cost of producing Mountain Drool in Massachusetts be $C_{m,m}$, etc. Because of differential staffing, it takes different amounts of work to make the product

at each plant, $w_{m,k}$ to make a unit of Mountain Drool in Kentucky and so on. Each factory has a maximum number of work units it can put in: m_k and m_m. To fill the insatiable maws of the public with delightful carbonated beverages, PrepsiCo must produce a total of g_p units of Prepsi Cola and g_m units of Mountain Drool. They would like to produce all the necessary product at the minimum total cost.

Formulate this problem as a linear programming instance. You do not have to *solve* the linear program (which would be difficult, as I haven't given you any actual numbers – PrepsiCo is pretty tightfisted with their trade secrets), but you do have to write the optimization criteria and boundary conditions. What is the dimension of the optimization (i.e., search) space for this problem? How many hyperplanes are expressed by the constraints?

The key to this problem is to recognize that all of the quantities I have defined in the problem are constants*. In order to represent the* amount *of each product produced at each plant, you need to introduce the* variables *$x_{p,k}$ (the amount of Prepsi produced in Kentucky), $x_{m,m}$ (amount of Drool produced in Massachusetts), etc.*

With those variables, you should see that if it costs $C_{p,k}$ to produce one unit of Prepsi in Kentucky and the KY plant is making a total of $x_{p,k}$ units of Prepsi, then it is spending $x_{p,k}C_{p,k}$ total in Prepsi production. Therefore, your total cost function is:

$$\text{Minimize: } x_{p,k}C_{p,k} + x_{m,k}C_{m,k} + x_{p,m}C_{p,m} + x_{m,m}C_{m,m}$$

To describe the constraints of the system, first you need to realize that every constraint must contain some variable *(i.e., one of the $x_{i,j}$). An expression that contains only constants (such as $w_{p,k} \leq m_k$) is either trivially true or impossible to satisfy. Remember that the only things you have control over in the LP solution process are the variables – the constants are established by the external world, economic forces, etc., and you don't have the power to change them. Your constraints are really telling you what values of your variables are or are not legitimate. Thus:*

$$\begin{array}{rcll}
\text{Subject to:} & & & \\
x_{p,k} & \geq & 0 & \text{(nonnegativity constraint)} \\
x_{m,k} & \geq & 0 & \\
x_{p,m} & \geq & 0 & \\
x_{m,m} & \geq & 0 & \\
x_{p,k}w_{p,k} + x_{m,k}w_{m,k} & \leq & m_k & \text{(total work constraint)} \\
x_{p,m}w_{p,m} + x_{m,m}w_{m,m} & \leq & m_m & \\
x_{p,k} + x_{p,m} & \geq & g_p & \text{(total production constraint)} \\
x_{m,k} + x_{m,m} & \geq & g_m &
\end{array}$$

The first four constraints say that you can't produce a negative amount of any product at any factory. The total work constraints say that neither factory can spend more effort than its maximum allowable workload, and the total production constraints say that you have to end up producing at least the minimum number of units of each soda.

Note that there are four variables in this system ($x_{i,j}$) so the dimension of the optimization problem is 4; there are eight constraints and each constraint specifies a single hyperplane, so the total number of hyperplanes is 8.