

## Exercise: English $\equiv$ FOPL?

Solutions...

Translate the following expressions from English into the P.C., and identify whether the result is a term, atom, sentence, etc., and also whether it's ground or closed.

- The One Ring.

**A:** ring.

*The One Ring is a single object. We're not asserting anything about it, so it remains just a constant. This is a constant and a term, but not an atom or a sentence (it doesn't have a truth value, it simply is).*

- Sam doesn't like Gollum.

**A:**  $\neg$ likes(sam,gollum).

*This is just an atom which asserts that it is not the case that sam (a unique object in the world) likes gollum (another unique object in the world). It is a grounded sentence b/c there are no variables – only constants.*

- Sam likes stewed rabbit.

**A:**  $\forall X$  stewed(X)  $\wedge$  rabbit(X)  $\rightarrow$  likes(sam,X)

*This is fundamentally different than the previous. Rabbits are hardly unique objects in the world, nor are all rabbits stewed. This just says that for every object which is both stewed and a rabbit, Sam likes it. This is a closed sentence, but not a grounded one (every variable is quantified, but there are still variables – it isn't all just about constants).*

- All dragons love treasure.

**A:**  $\forall X \forall Y$  dragon(X)  $\wedge$  treasure(Y)  $\rightarrow$  loves(X,Y)

*This says that for all things in the world, X and Y, if X is a dragon and Y is a treasure, then X loves Y. Note that this does not commit to there actually being any dragons or treasures. All it says is if they exist, then the dragon loves the treasure.*

- Every dragon has a favorite treasure.

**A:** *This one is a lot trickier. One way to say it is this:*

$$\forall X \text{ dragon}(X) \rightarrow \exists Y (\text{treasure}(Y) \wedge \text{favorite\_of}(X, Y))$$

*That claims that for every dragon (possibly a null set), there must be at least one thing Y such that Y is the favorite of the dragon. A bit more complex way to say it is to start with:*

$$\forall X \text{ dragon}(X) \rightarrow \exists Y (\text{treasure}(Y) \wedge \text{loves}(X, Y))$$

*This looks good and is what we first came up with in class, but it's problematic. As one of your colleagues pointed out, it doesn't restrict there to be a single favorite (presumably the predicate favorite\_of, above, encoded that directly). What we need is some way to specify that Y is unique or is the best. One way is to extend FOPL to have a "exists only one" operator. Another solution is to say that there isn't any other treasure that the dragon loves more:*

$$\forall X \text{ dragon}(X) \rightarrow \exists Y (\text{treasure}(Y) \wedge \forall Z \neg (\text{treasure}(Z) \wedge \text{loves\_more}(X, Z, Y)))$$

*Again, this is a closed, but not grounded, sentence.*

- When Frodo wears the Ring, then the Nazgûl can see him.

*Note that Nazgûl is plural.*

**A:** wears(frodo,ring)  $\rightarrow$  ( $\forall X$  nazgul(X)  $\rightarrow$  can\_see(X,frodo))

*Again, a closed sentence. Notice that this requires cascaded implication.*

- There is a fire hot enough to destroy the Ring.

**A:**  $\exists X$  fire(X)  $\wedge$  greater(temperature(X),melting\_point(ring))

*This is pretty straightforward – it's just a closed sentence that asserts that there is at least one fire whose temperature (a unary function) is greater than (a binary function) the melting point (a unary function) of the ring (a constant).*