Quiz 0: Solutions

6. Give the expression for the pdf of a 1-dimensional Gaussian random variable x with mean μ and variance σ

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{(x-\mu)^2}{2\sigma^2}}$$

7. Give the expression for the pdf of a N-dimensional Gaussian random variable X with mean $\bar{X}$ and variance Σ

$$f(X) = \frac{1}{(2\pi)^{N/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(X-\bar{X})^T\Sigma^{-1}(X-\bar{X})}$$

8. Given the following distribution over three discrete random variables X, Y, and Z, each taking on the values 0 or 1:

$X = 0$

	$Z = 0$	$Z = 1$
$Y = 0$	0	$\frac{1}{12}$
$Y = 1$	$\frac{1}{4}$	$\frac{1}{4}$

$X = 1$

	$Z = 0$	$Z = 1$
$Y = 0$	$\frac{1}{12}$	$\frac{1}{12}$
$Y = 1$	0	$\frac{1}{4}$

Find:

(a) The joint distribution of Y and Z:

$P(Y, Z) =$

	$Z = 0$	$Z = 1$
$Y = 0$	$\frac{1}{12}$	$\frac{1}{6}$
$Y = 1$	$\frac{1}{4}$	$\frac{1}{2}$

(b) The marginal distributions of Y and Z:

$P(Y) =$

$Y = 0$	$\frac{1}{4}$
$Y = 1$	$\frac{3}{4}$

$P(Z) =$

$Z = 0$	$\frac{1}{3}$
$Z = 1$	$\frac{2}{3}$

(c) The conditional distribution of X given that $Y = 0$: $P(X|Y) =$

$X = 0$	$\frac{1}{3}$
$X = 1$	$\frac{2}{3}$

(d) Are X and Y independent? *NO*

(e) Are Y and Z independent? *YES (note that $P(Y, Z) = P(Y)P(Z)$)*

9. **Define:**

(a) Degree of a graph: *Maximum number of arcs leaving any node*

(b) Branching factor of a tree: *Maximum number of arcs leaving any node (tricky, eh?)*

(c) NP-Complete: *A problem is NP-complete if (a) there is a polynomial time way to verify any proposed solution (polynomial certificate) and (b) the problem is at least as hard as any other problem in NP (polynomial reducability). Alternatively, the problem can be solved in polynomial time on a nondeterministic turing machine.*

(d) PSPACE-Complete: *A problem is PSPACE complete if it can be solved in at most a polynomial amount of* space *and is at least as hard as any other problem in PSPACE (i.e., any other PSPACE problem can be polynomially reduced to it.)*

(e) Markov property: *A random process is Markov if the statistical distribution of the future outcomes depend only on a fixed number of previous outcomes (typically 1). Alternatively:*

$$\Pr[x_{t+1}|x_t, x_{t-1}, \ldots, x_1] = \Pr[x_{t+1}|x_t, x_{t-1}, \ldots, x_{t-k}] \text{ for some fixed } k < t$$

10. **True or false:** A (nontrivial) positive definite matrix posesses an equal number of real and imaginary eigenvalues. ***FALSE*** *A positive definate matrix posesses only positive real eigenvalues.*