Proof Sketches Discussion

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Hierarchy of Theories

Let A_i be an independent set of axioms for theory $\mathcal{T}_i$.

Say have $A_1 \subset A_2 \subset A_3$ and assume same inference rules.

$\Rightarrow \mathcal{T}_1 \subset \mathcal{T}_2 \subset \mathcal{T}_3$ (theorems)

Example: lattice hierarchy

$\mathcal{LT} \subset \mathcal{CL} \subset \mathcal{OL} \subset \mathcal{OML} \subset \mathcal{MOL} \subset \mathcal{BA}$

Can knowledge about theories $\mathcal{T}_1$ and $\mathcal{T}_3$ help us prove theorems in theory $\mathcal{T}_2$?

Two ideas about picking given clauses

- semantics (consider models in $\mathcal{T}_1$)
- proof sketches (consider proofs in $\mathcal{T}_3$)

Proof Sketches

Consider a derivation as a sequence of clauses,

$$C_1, C_2, ..., C_i, ..., C_j, ..., C_n$$

where

- C_i is an extra assumption wrt the target theory
- derived clause C_j has C_i in its derivation history

C_j either is in the target theory or it is not.

- if yes, it suffices to find a new derivation of C_j
- if not, it suffices to "bridge the gap"

In either case, we've reduced a large problem to a smaller (but still potentially difficult) problem.

Theory of Proof Sketches?

Is there something interesting going on here, or is this just an AR hacker's heuristic (that happens to have a great track record)?

- Operational view
- What makes a "good theorem"? Can we presume that there is some regularity and structure to known theories and to the results of interest?
- Probabilistic view? Of the space of all available clauses, with no other prior information, it seems reasonable to focus on clauses that have shown value previously.

Example

`http://www.cs.unm.edu/˜veroff/LOOPS/basarab.html`