IS QHA EQUATIONALLY DEFINABLE?

A *quasi-Hilbert algebra* is an algebra $\langle A; \to, 1\rangle$ of type $\langle 2, 0\rangle$ satisfying the following collection of identities and quasi-identities:

$$x \to x \approx \mathbf{1}$$

$$x \to y \approx \mathbf{1} \mathbin{\&} y \to x \approx \mathbf{1} \supset x \approx y$$

$$x \to (y \to x) \approx \mathbf{1}$$

$$x_1 \to (x_2 \to x) \approx \mathbf{1} \mathbin{\&} x_1 \to (x_2 \to (x \to y)) \approx \mathbf{1} \supset x_1 \to (x_2 \to y) \approx \mathbf{1}.$$

Clearly the class QHA of all quasi-Hilbert algebras is a quasivariety.

Problem: Is QHA a variety? If QHA is equationally definable, provide an axiomatisation.

By results of Bou *et al*, there is some algebraic evidence to suggest that QHA is a variety.

There has been no progress made on this problem since ADAM 2006.