CS 522: Digital Image Processing
Homework 1 (Fall ’11)

1 Theory

1. A p.d.f. for a continuous random variable $X$ is defined as follows:

$$f_X(x) = \begin{cases} 
2x/9 & \text{if } 0 < x < 3 \\
0 & \text{otherwise.}
\end{cases}$$

Find the value of the c.d.f. at 1:

$$F_X(1) = \int_0^1 f_X(x) \, dx.$$ 

2. Let $X$ and $Y$ be continuous random variables where $f_X(x) = \frac{1}{\tau} e^{-x/\tau}$ and let $Y = X^2$. Derive an expression for $f_Y$. Hint: Don’t forget that the inverse is not uniquely defined.

2 Practice

1. Write a function `cumulative-distribution-function` which takes an image as its argument and returns the discrete cumulative distribution function (c.d.f.) for the image:

$$F(j) = \frac{255}{nm} \sum_{i=0}^{j} H(i)$$

where $n$ is the number of rows, $m$ is the number of columns, and $H$ is the grey-level histogram. You may assume that the image contains grey-levels in the range $[0, 255]$. The c.d.f. should be returned as a `vector`. Compute the discrete c.d.f. for the `frog` image and for an image of your choice. Hint: Although not strictly necessary, learning the Scheme `do` macro might help you.

2. Write a function `histogram-equalize` which takes an image as its argument and returns an image which has been histogram equalized using the discrete c.d.f. as a grey-level transformation. Plot the histograms for the `frog` image and for an image of your choice before and after histogram equalization.
You should also show both images before and after histogram equalization. Hint: This is easy to do using image-map.

3. Write a function `inverse-cumulative-distribution-function` which takes an image as its argument and returns the discrete inverse cumulative distribution function (discrete i.c.d.f.) for the image. The value of the discrete i.c.d.f. $F^{-1}$ at $j$ is the minimum value $k$ such that $F(k) \geq j$. You may assume that grey-levels are in the range $[0, 255]$. The i.c.d.f. should be returned as a Scheme vector.

4. Write a function `histogram-match` which takes two images as its arguments and returns an image which is the result of applying the histogram matching grey-level transformation to the first image so that its histogram is matched to that of the second image. Plot the histograms for the frog image and for the cropped-rad image after its histogram has been matched to that of the frog image. Show the transformed cropped-rad image. Repeat the above for two equal sized images of your choice. Plot the histograms and show the images before and after (for the second image) histogram matching. Hint: This is easy to do using image-map.