1 Theory

1. A p.d.f. for a discrete random variable $X$ is defined as follows:

$$f_X(x) = \begin{cases} 2x/9 & \text{if } 0 < x < 3 \\ 0 & \text{otherwise.} \end{cases}$$

Find the value of the c.d.f. at 1:

$$F_X(1) = \int_0^1 f_X(x) dx.$$ 

2. Let $X$ and $Y$ be continuous random variables where $f_X(x) = \frac{1}{\tau} e^{-x/\tau}$ and let $Y = X^2$. Derive an expression for $f_Y$. Hint: Don’t forget that the inverse is not uniquely defined.

2 Practice

1. Write a function `cumulative-distribution-function` which takes an image as its argument and returns the discrete cumulative distribution function (c.d.f.) for the image:

$$F(j) = \frac{255}{nm} \sum_{i=0}^{j} H(i)$$

where $n$ is the number of rows, $m$ is the number of columns, and $H$ is the grey-level histogram. You may assume that the image contains grey-levels in the range $[0, 255]$. The c.d.f. should be returned as a vector. Compute the discrete c.d.f. for the `frog` image and for an image of your choice. Hint: Although not strictly necessary, learning the Scheme `do` macro might help you.

2. Write a function `histogram-equalize` which takes an image as its argument and returns an image which has been histogram equalized using the discrete c.d.f. as a grey-level transformation. Plot the histograms for the `frog` image and for an image of your choice before and after histogram equalization.
You should also show both images before and after histogram equalization. Hint: This is easy to do using \textit{image-map}.

3. Write a function \textit{inverse-cumulative-distribution-function} which takes an image as its argument and returns the discrete inverse cumulative distribution function (discrete i.c.d.f.) for the image. The value of the discrete i.c.d.f. $F^{-1}$ at $j$ is the minimum value $k$ such that $F(k) \geq j$. You may assume that grey-levels are in the range $[0, 255]$. The i.c.d.f. should be returned as a Scheme \texttt{vector}.

4. Write a function \textit{histogram-match} which takes two images as its arguments and returns an image which is the result of applying the histogram matching grey-level transformation to the first image so that its histogram is matched to that of the second image. Plot the histograms for the \textit{frog} image and for the \textit{cropped-rad} image after its histogram has been matched to that of the \textit{frog} image. Show the transformed \textit{cropped-rad} image. Repeat the above for two equal sized images of your choice. Plot the histograms and show the images before and after (for the second image) histogram matching. Hint: This is easy to do using \textit{image-map}.