1. Characterize the following as linear, or non-linear, and as even, odd, or neither, and prove your characterization. In these functions, \( x \) is real, and \( z \) is complex.

(a) \( f(x) = 7x + 1 \)
(b) \( f(x) = x^2 - x \)
(c) \( f(x) = e^{-x^2} \)
(d) \( f(z) = 3z - 2 \)
(e) \( f(z) = z - z^* \)
(f) \( f(z) = \text{Im}(z) \)
(g) \( f(z) = z^{1/2} \)

2. The \( \int \) operator takes a function, \( f \), as its argument and returns the antiderivative of the function: \( f \rightarrow \int f(t) \, dt \). Prove that the \( \int \) operator is:

(a) Linear.
(b) Shift-invariant.

3. Prove that \( \sin(x) = \frac{e^{jx} - e^{-jx}}{2j} \).

4. The impulse response function of a linear, shift-invariant system is:

\[
 h(t) = \frac{\sin(\pi t)}{\pi t}
\]

and its input is:

\[
 x(t) = \cos(4\pi t) + \cos(\pi t/2).
\]

What is its output?

5. The impulse response function of a linear, shift-invariant system is:

\[
 h(t) = e^{-\frac{\pi t^2}{2}}
\]

and its input is:

\[
 x(t) = e^{j2\pi s_0 t}.
\]

What is its output?
6. The sine Gabor function is the product of a sine and a Gaussian, \( f(t) = e^{-\pi t^2} \sin(2\pi s_0 t) \). Give an expression for \( F(s) \), the Fourier transform of \( f(t) \).

7. Prove that the sum of two Gaussian random variables with variances \( \sigma_1^2 \) and \( \sigma_2^2 \) is a Gaussian random variable with variance \( \sigma_1^2 + \sigma_2^2 \).

8. The function, \( f(t) \), is defined as:

\[
\begin{align*}
    f(t) &= \begin{cases} 
        1 & \text{if } |at - b| \leq \frac{1}{2} \\
        0 & \text{otherwise} 
    \end{cases} 
\end{align*}
\]

Give an expression for \( F(s) \), the Fourier transform of \( f(t) \).

9. The transfer function of a linear shift invariant system is \( H(s) = 1/s \). The impulse response function, \( h(t) \), is \( \mathcal{F}^{-1}\{H(s)\} \). Give an expression for \( g(t) \) where:

\[
g(t) = \int_{-\infty}^{\infty} e^{j2\pi s_0 \tau} h(t - \tau) \, d\tau.
\]

10. Compute the Fourier transform of \( f(t) = -2\pi t \, e^{-\pi t^2} \cos(2\pi s_0 t) \). Hint: What is \( \frac{d(e^{-\pi t^2})}{dt} \)?

11. Prove the following statement: If \( \mathcal{F}\{f\}(s) = F(s) \) then \( \mathcal{F}\{F\}(s) = f(-s) \). Hint: If \( \mathcal{F}\{f\}(s) = F(s) \) then \( \mathcal{F}^{-1}\{F\}(t) = f(t) \).

12. Prove that \( \mathcal{F}^{-1}\{\mathcal{F}\{f\}\} = f \)