CS 422: Digital Image Processing
Homework 5 (Spring '11)

1. (4 pts.) Let X and Y be discrete random variables with probability mass functions $p_x(x_i)$ and $p_y(y_j)$ and joint probability mass function $p_{XY}(x_i, y_j)$. Give a necessary and sufficient condition for X and Y to be statistically independent.

2. (2 pts.) Which of the following is the expression for the *convolution* of f and g:

   (a) $\{f * g\}(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)dt$

   (b) $\{f * g\}(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$

   (c) $\{f * g\}(t) = \int_{-\infty}^{\infty} f(t)g(t-\tau)dt$

   (d) $\{f * g\}(t) = \int_{-\infty}^{\infty} f(t)g(t+\tau)dt$

   (e) $\{f * g\}(t) = \int_{-\infty}^{\infty} f(\tau)g(t+\tau)d\tau$

3. (7 pts.) Let $\mathbf{x} = [1\ 2\ 3\ 4\ 5]^{\mathrm{T}}$ and $\mathbf{y} = [1\ 0\ 0\ 0\ 1]^{\mathrm{T}}$. Give values for the following:

   (a) $\mathbf{x} * \mathbf{y}$ where $\mathbf{x} * \mathbf{y}$ is the discrete periodic convolution of $\mathbf{x}$ and $\mathbf{y}$

   (b) the matrix $\mathbf{C}$ such that $\mathbf{x} * \mathbf{y} = \mathbf{C}\mathbf{x}$.

4. (7 pts.) For two complex numbers, $z_1 = a_1 \cos\theta_1 + a_1 j \sin\theta_1$ and $z_2 = a_2 \cos\theta_2 + a_2 j \sin\theta_2$, give expressions for:

   (a) the complex conjugate of z_1, *i.e.*, z_1^*

   (b) z_1 in polar form

   (c) the complex conjugate of z_1 in polar form

   (d) $z_1 + z_1^*$

   (e) $z_1 z_1^*$

   (f) $z_1 + z_2$

   (g) $z_1 z_2$.

5. (2 pts.) True or False. $f * g = g * f$.

6. (2 pts.) True or False. A linear grey scale transformation can only stretch or compress the histogram and shift it right or left.

7. (2 pts.) True or False. The cumulative distribution function (cdf) is the grey scale transformation which flattens the histogram.

8. (2 pts.) True or False. The inverse cumulative distribution function (icdf) is the grey scale transformation which flattens the histogram.

9. (4 pts.) A gray scale image is represented using one byte per pixel. Give a transformation which will make dark pixels light and light pixels dark. What is the effect of this transformation on the histogram?

10. (2 pts.) True or False. The discrete Fourier transform matrix is unitary and symmetric.

11. (2 pts.) True or False. The discrete Fourier transform matrix is its own inverse.

12. (2 pts.) True or False. The convolution of a harmonic signal of frequency, s, with any function is a harmonic signal of frequency, s.

13. (2 pts.) True or False. The convolution of a Gaussian with a harmonic signal is a Gaussian.

14. (2 pts.) True or False. In the discrete case, a linear shift invariant system can always be characterized as multiplication by an orthonormal matrix.

15. (2 pts.) True or False. If a system produces output, $y(t)$, when given input, $x(t)$, and output, $y(t+\tau)$, when given input, $x(t+\tau)$, then the system is linear.

16. (2 pts.) True or False. The Fourier transform of the convolution of two real even functions is imaginary and odd.

17. (2 pts.) True or False. In binary image morphology, the complement of the erode of the complement of a binary image is the open of the binary image.

18. (2 pts.) True or False. In binary image morphology, the complement of the dilate of the complement of a binary image is the erode of the binary image.

19. (6 pts.) Give definitions for the morphological operations open and close in terms of the operations dilate and erode.

20. (6 pts.) Give a precise (*i.e.,* mathematical) statement of the conditions which must hold between two spectral distributions, $C_1(\lambda)$ and $C_2(\lambda)$, if they are metamers.

21. (2 pts.) True or False. In human color vision it is impossible to reproduce the color of a pure spectral source as convex combination of pure spectral sources.

22. (2 pts.) True or False. In the transformation from tristimulus values to chromatic coordinates information about a color's saturation is lost.

23. (2 pts.) True of False. In the transformation from tristimulus values to chromatic coordinates information about a color's intensity is lost.

24. (12 pts.) Let $S_\ell(\lambda)$, $S_m(\lambda)$, and $S_s(\lambda)$ be the spectral sensitivity functions of the long, medium, and short wavelength cones of the human retina, let $C_r(\lambda) = \delta(\lambda - 700 \text{ nm})$, $C_g(\lambda) = \delta(\lambda - 546 \text{ nm})$, and $C_b(\lambda) = \delta(\lambda - 436 \text{ nm})$, be the three CIE standard light sources and let $C(\lambda)$ be the spectral distribution of a flower.

    (a) Give expressions for x, y, and z, the tristimulus values of the flower's color.

    (b) Give expressions for X and Y, the chromatic coordinates of the flower's color.

    (c) Give a system of linear equations, which when solved, gives the amounts, $V_r(C)$, $V_g(C)$, and $V_b(C)$, of the three CIE standard light sources necessary to reproduce the color of the flower.

25. (4 pts.) Describe the output of the distance transform of a binary image.

26. (8 pts.) Show that the Fourier transform of $\cos 2\pi t$ is equal to $\frac{1}{2}[\delta(t-1) + \delta(t+1)]$.

27. (2 pts.) What is $f(t) * \delta(t - t_0)$?

28. (2 pts.) What is $[f(t) + g(t)] * \delta(t - t_0)$?

29. (2 pts.) What is $\int_{-\infty}^{\infty} \delta(t)dt$?

30. (2 pts.) What is $\int_{-\infty}^{\infty} |a|\delta(at)dt$?

31. (6 pts.) Let $F(s) = \int_{-\infty}^{\infty} e^{-j2\pi st} f(t)\, dt$. What is $\int_{-\infty}^{\infty} e^{j2\pi s\tau} f(t-\tau)\, d\tau$?

32. (4 pts.) What is the value of the expression $\int_{-\infty}^{\infty} e^{j2\pi s_0 t} e^{-j2\pi s_1 t} dt$?

33. (4 pts.) What is the value of $\int_{-\infty}^{\infty} e^{j2\pi st} \left[\int_{-\infty}^{\infty} e^{-j2\pi st} f(t)\, dt\right]\, ds$?

34. (4 pts.) Let $H_I(i)$ be the histogram of an image with n rows and m columns. If grey values are represented using 8 bits per pixel, what is the value of the expression $\sum_{i=0}^{255} H_I(i)$?

35. (4 pts.) Let $H_{RGB}(r, g, b)$ be the three dimensional joint histogram of a color image with n rows and m columns. Let R, G and B be the three color component images. If the values of the pixels of the color-component images are represented using 8 bits per pixel, what is the value of the expression $\sum_{r=0}^{255} \sum_{g=0}^{255} \sum_{b=0}^{255} H_{RGB}(r, g, b)$?

36. (4 pts.) Let $H_{RGB}(r, g, b)$ be a three dimensional joint histogram of a color image. Let R, G and B be the three color-component images. Let the values of the pixels of these images be represented using 8 bits per pixel. Give an expression for $H_R(r)$, the histogram of the red color-component image.

37. (4 pts.) Give an expression for the orientation of the harmonic signal $e^{j2\pi(ux+vy)}$ where u and v are the spatial frequencies in the x and y direction.