1. Consider the following advection-diffusion partial differential equation:
\[ \frac{\partial P}{\partial t} = -C \frac{\partial P}{\partial x} + D \frac{\partial^2 P}{\partial x^2}. \]

(a) Give finite difference approximations for \( \frac{\partial P}{\partial t} |_{x,t} \), \( \frac{\partial P}{\partial x} |_{x,t} \), and \( \frac{\partial^2 P}{\partial x^2} |_{x,t} \).
(b) Give an expression for \( P(x, t + \Delta t) \) in terms of \( P(x, t) \), \( P(x + \Delta x, t) \), and \( P(x - \Delta x, t) \).

2. A bivariate Gaussian random variable, \( x = [x_0 \ x_1]^T \), has the following p.d.f.:
\[ f(x_0, x_1) = \left(\frac{a^2 - b^2}{2\pi}\right)^{\frac{1}{2}} \exp\left(-\frac{1}{2} [ax_0x_0 + 2bx_0x_1 + ax_1x_1]\right). \]

(a) Give the matrix \( W \) which will decorrelate the components of \( x \).
(b) Let \( u = Wx \). Give an expression for \( g(u_0, u_1) \), the p.d.f. for the bivariate Gaussian random variable, \( u = [u_0 \ u_1]^T \).

3. Let \( S_r(\lambda), S_g(\lambda), \) and \( S_b(\lambda) \) be the spectral sensitivity functions of the red, green, and blue cones of the human retina. Let \( C(\lambda) \) be the spectral distribution of a sunlit daffodil. Define a system of linear equations, which when solved, gives the amounts, \( V_r(C), V_g(C), \) and \( V_b(C) \), of the three CIE standard primary sources, \( \delta(\lambda - 700), \delta(\lambda - 546), \) and \( \delta(\lambda - 436) \), necessary to reproduce the color of the sunlit daffodil.

4. The \( n \)-th moment of \( \Psi \) is defined to be \( M_n\{\Psi\} = \int_{-\infty}^{\infty} t^n \Psi(t) dt \). Let \( f(t) = e^{-\pi t^2}, f'(t) = -2\pi t e^{-\pi t^2}, \) and \( f''(t) = 2\pi e^{-\pi t^2}(2\pi t^2 - 1) \). Prove the following:

(a) \( M_0\{f'\} = 0 \).
(b) \( M_0\{f''\} = M_1\{f''\} = 0 \).

5. The six vectors, \( f_1 = [\cos(\pi/3) \ \sin(\pi/3)]^T, f_2 = [\cos(\pi/3) \ -\sin(\pi/3)]^T, f_3 = [-1 \ 0]^T, f_4 = [-\cos(\pi/3) \ -\sin(\pi/3)]^T, f_5 = [-\cos(\pi/3) \ \sin(\pi/3)]^T, \) and \( f_6 = [1 \ 0]^T \) form a frame \( \mathcal{F} \) for \( \mathbb{R}^2 \). Draw the frame.
(a) Give two representations for the vector, \( x = \begin{bmatrix} 1 & 1 \end{bmatrix}^T \), in \( F \).

(b) Prove that \( x \) has an infinite number of representations in \( F \).

(c) Give a matrix which transforms any representation of a vector in \( F \) into its representation in the standard basis for \( \mathbb{R}^2 \).

(d) Give a matrix which transforms a representation of any vector in the standard basis for \( \mathbb{R}^2 \) into its representation in \( F \).