11.1 \((m + 1)^3\)

11.3 As \(u\) varies over \((a, b)\), \(v = \frac{b - u}{b - a}\) varies over \((0, 1)\). Substituting into the polynomial \(p(u) = \sum_{k=0}^{n} c_k u^k\), we have \(q(v) = \sum_{i=0}^{n} d_i v^i = \sum_{k=0}^{n} c_k ((b - a)v + a)^k\). We can expand the products on the right and match powers of \(v\) to obtain \(\{d_i\}\).

11.5 Consider the Bernstein polynomial

\[
\begin{align*}
b_{kd}(u) &= \binom{d}{k} u^k (1 - u)^{d-k}.
\end{align*}
\]

For \(k = 0\) or \(k = d\), the maximum value of 1 is at one end of the interval \((0, 1)\) and the minimum is at the other because all the zeros are at 1 or 0. For other values of \(k\), the polynomial is 0 at both ends of the interval and we can differentiate to find that the maximum is at \(u = k/d\). Substituting into the polynomial, the maximum value is \(\frac{d!}{k!(d-k)!} \frac{k^k}{(d-k)!}\), which is always between 0 and 1.

11.7 Any quadric can be written as

\[
q(x, y, z) = ax^2 + by^2 + cz^2 + 2dxy + 2exz + 2fyz + 2gx + 2hy + 2iz + j = 0,
\]

where \(a, b, c, d, e, f, g, h, i\) and \(j\) are constants. Let \(p^T = \begin{bmatrix} x & y & z & 1 \end{bmatrix}\).

Then, we can rewrite the equation as

\[
q(p) = p^T Q p = 0,
\]

where

\[
Q = \begin{bmatrix}
a & d & e & g \\
d & b & f & h \\
e & f & c & i \\
g & h & i & j
\end{bmatrix}.
\]

Note that we can also use \(p^T = \begin{bmatrix} x & y & z & w \end{bmatrix}\) where \(w\) can be any constant.

11.15 For \(r = 0\) we get the line between \(P_0\) and \(P_2\). For \(r = \frac{1}{2}\) we get the parabola \(u^2 P_0 + 2u(1-u)P_1 + (1-u)^2 P_2\) which passes through \(P_0\) and \(P_2\).
For $r > \frac{1}{2}$, we obtain hyperbolas, and for $r < \frac{1}{2}$, we obtain ellipses. Thus, we can use NURBS to obtain both parametric polynomial curves and surfaces, and to obtain quadric surfaces.

11.17 We can write the Hermite surface as

$$p(u, v) = u^T M_H Q M_H^T v = u^T A v,$$

where $Q$ contains the control point data and $M_H$ is the Hermite geometry matrix. If evaluate $p, \frac{\partial p}{\partial u}, \frac{\partial p}{\partial v}$, and $\frac{\partial^2 p}{\partial u \partial v}$ at the corners we find that the 16 values in the matrix $A$ are the 4 values at the 4 corners of the patch, the first partial derivatives $\frac{\partial p}{\partial u}$ and $\frac{\partial p}{\partial v}$ at the corners and the first mixed partial derivative $\frac{\partial^2 p}{\partial u \partial v}$ at the corners.

11.19 This process creates a quadric curve which interpolates $P_0$ and $P_2$ and lies in the triangle defined by $P_0, P_1,$ and $P_2$.

11.21 Nothing unusual happens other than the slope at $u = 0$ must be zero as long as the control points are still separated in parameter space.

11.25 The columns of the matrix $M_R$ contain the coefficients of the blending polynomials which are

$$p_0(u) = -u^3 + 2u^2 - u,$$
$$p_1(u) = 2u^3 - 5u^2 + 2,$$
$$p_2(u) = -3u^3 + 4u^2 - u,$$
$$u^3 - u^2.$$

Note that the third and fourth polynomials can be obtained from the first and second by substituting $1 - u$ for $u$. We zeros of the fourth polynomial are 0, 0, and 1 so the zeros of the first are 0, 1, and 1. We can obtain the zeros of the third by factoring out $u$ which gives a zero at 0 and solving the resulting quadratic equation to find the zeros at $\frac{-3 \pm \sqrt{7}}{2}$. The zeros of the second polynomial are thus 1 and $\frac{1 \pm \sqrt{7}}{2}$.