Homework set 4: Grammars, regular languages, and context-free languages — due Monday 5 February

Total number of points available on this homework is 150. Full credit is equivalent to 100 points.

1. (20 pts.) What is the language generated by the grammar \( S \rightarrow SSS | a \)?

2. (20 pts.) Write a grammar for the language of the regular expression \(((a+b^*c)d)^*\).

3. (30 pts.) Give examples of languages that satisfy the following conditions:
   
   (a) \( L_1 \) is regular and infinite, \( L_2 \) is not regular, and \( L_1 \subseteq L_2 \).
   
   (b) \( L_3 \) is regular, \( L_4 \) is not regular, and \( L_4 \subseteq L_3 \).

4. (30 pts.) The reverse of a string \( x \) is denoted \( x^R \), and is defined recursively as follows: \( \varepsilon^R = \varepsilon \), and for \( \sigma \in \Sigma \), \( (\sigma x)^R = \sigma x^R \).

   Consider the following grammar \( G \) over \( \Sigma = \{a, b\} \):
   
   \[
   S \rightarrow aSa \\
   S \rightarrow bSb \\
   S \rightarrow \varepsilon
   \]

   Prove that it is possible to derive from \( S \) any string of the form \( xSx^R \), where \( x \) is any string over \( \{a, b\} \).

5. (50 pts. - extra credit) Show that English is not context-free.

If you wish to study formal languages beyond the brief introduction we were able to do in class, here are some good books, ordered from more theoretical to more applied: