Homework set 9: \( \lambda \)-calculus — due Monday 5 March

Total number of points available on this homework is 100. Full credit is equivalent to 100 points.

1. (30 pts.) Show that the following terms have a normal form:
   (a) \((\lambda y. yyy)((\lambda a. b) I) (SS)\)
   (b) \((\lambda z. zy)((\lambda x. xxx)(\lambda x. xxx))(\lambda w. I)\)
   (c) \(SSSSSSSS\)

2. (40 pts.) For each of the following \( \lambda \)-expressions either find its normal form or show that it has no normal form:
   (a) \((\lambda x. xx)(\lambda x. x)\)
   (b) \((\lambda x. xx)(\lambda x. xx)\)
   (c) \(Y\) (see below)
   (d) \(Y(\lambda y. y)\)

3. (30 pts.) A \( \lambda \)-expression \textbf{Fix} with the property that \( \text{Fix} E = E(\text{Fix} E) \) for any \( E \) is called a fixed-point operator (or combinator). One well known fixed-point operator is \( Y \), defined as \( Y \equiv \lambda f. (\lambda x. f(xx))(\lambda x. f(xx)) \). Show that \( Y \) is a fixed-point operator.