Homework 2 — Second set of ML core language programs — assigned Wednesday 5 February, due Wednesday 12 February

Total number of points available on this homework is 180. Full credit is equivalent to 100 points.

Reading assignment

Read Chapter 4 of *ML for the Working Programmer*.

Read the descriptions of the following structures in the Standard ML Basis Library documentation: List, ListPair.

Optional reading: Read Chapters 1–9 of *The Little MLer*.

2.1 Map (10pts)

If \( xs \) is a list, the evaluation of \( \text{map } f \ (\text{map } g \ xs) \) requires two list traversals. Simplify the expression so that only one traversal is needed.

2.2 Simple expression evaluator (15pts)

We can use the following data type declaration to introduce a language of simple arithmetic expressions:

```ml
datatype expr = Num of int
               | Add of expr * expr
               | Mul of expr * expr
```

Write a function \( \text{eval} \), with type \( \text{expr} \rightarrow \text{int} \), which returns the arithmetic value of an expression.

2.3 Simple parser (30pts)

Write a function \( \text{parse} \), with type \( \text{string} \rightarrow \text{expr} \), where \( \text{expr} \) is as in the preceding exercise. The function should return an expression corresponding to the text of the string, in accordance with the standard precedence of arithmetic. The input string may consist of digits, + and * signs, parentheses, and white space. For instance, \( \text{parse } "5 + 3 * 4" \) should evaluate to \( \text{Add } (\text{Num } 5, \text{Mul } (\text{Num } 3, \text{Num } 4)) \). In case the input string is not well-formed (for example, the string \( "5 + 3 ) * 4" \) is not), function \( \text{parse} \) should raise an exception \( \text{Failed} \) because (see p. 135 of *ML for the Working Programmer*).
2.4 Using lists for arithmetic (45pts)

This is a continuation of exercise 1.5.

Write the following functions:

- **(15pts)** \texttt{mulLongInts: int \rightarrow (int list * int list) \rightarrow int list}, such that \texttt{mulLongInts r (a, b)} computes the product of the nonnegative integers given by lists \(a\) and \(b\); all lists use the same radix \(r\). Lists \(a\) and \(b\) can represent arbitrarily large positive integers.

- **(30pts)** \texttt{divLongInts: int \rightarrow (int list * int list) \rightarrow (int list * int list)}, such that \texttt{divLongInts r (a, b)} computes the quotient and the remainder from the division of the nonnegative integers given by lists \(a\) and \(b\); all lists use the same radix \(r\). Lists \(a\) and \(b\) can represent arbitrarily large positive integers. You can assume that \(b\) does not represent zero.

2.5 Tic-tac-toe (40pts)

The game board for the game of tic-tac-toe is shown in Figure 1. The two players, X and O, alternate in claiming the spaces on the board; X goes first. The game ends when all the spaces have been claimed or as soon as a player has claimed three spaces in a line, in which case that player has won the game. How many possible games of tic-tac-toe are there? First consider this question mathematically. Then write an ML program to produce the list of all possible games. Represent a game as a list of integers; for example, \([5, 2, 3, 7, 4, 6, 9, 1, 8]\) stands for the game in which X played the center (space 5), O played space 5, X played space 3, O played space 7, X played space 4, O played space 6, X played space 9, O played space 1, X played space 8, and the game ended in a draw.

\begin{center}
\begin{tabular}{|c|c|c|}
\hline
1 & 2 & 3 \\
\hline
4 & 5 & 6 \\
\hline
7 & 8 & 9 \\
\hline
\end{tabular}
\end{center}

Figure 1: The tic-tac-toe game board.
2.6 Integrating functions (40pts)

Write a higher-order function

\[
\text{numericallyIntegrate} : \text{real} \to (\text{real} \to \text{real}) \to (\text{real} \to \text{real}) \to \\
(\text{real} \times \text{real}) \to \text{real}
\]

such that

\[
\text{numericallyIntegrate} \epsilon f f' (a, b)
\]

computes (an approximation to) the integral of the real function \( f \), the derivative of which is given by \( f' \), over the real interval \((a, b)\). The quantity \( \epsilon \) expresses the desired accuracy of the approximation; it is up to you to interpret this quantity according to the integration algorithm you decide to use.

How to turn in

Turn in your code by running

\`
\text{roshan/handin your-file}
\`

on a regular UNM CS machine.

You should use whatever filename is appropriate in place of your-file. You can put multiple files on the command line, or even directories. Directories will have their entire contents handed in, so please be sure to clean out any cruft.