1 Image Watermarking

1. Using the non-watermarked image rove as reference, recover the watermark from the image watermarked-rove. Both images are stored in ASCII .pgm format with 12 bit integer grey values and can be downloaded from the class webpage. Now save both images in ASCII .pgm format with 8 bit integer grey values (the default) and repeat the operations you previously used to recover the watermark.

2. Write a function make-watermark which takes a square image, image, an integer, n, and a float epsilon, as arguments. You may assume that the image is of size $2^m$ (for integer $m$) and has grey values in the range $[0, 255]$. The function returns a floating point image representing a watermark of size $2^{m+n}$, with grey values in the range $[1 - \epsilon, 1 + \epsilon]$, constructed recursively according to the scheme shown in Figure 1. Construct a watermark using an image of your choice using a value of $n \geq 2$.

3. Write a function insert-watermark, which takes two square images of equal size ($2^m$ for integer $m$) representing an input image image and a watermark mark as arguments, and returns image watermarked with mark. Use your function to watermark an image of your choice with a watermark of your choice. Save your image in ASCII .pgm format with 8 bit integer grey values.

4. Write a function recover-watermark, which takes two square images of equal size ($2^m$ for integer $m$), representing a watermarked image, marked – image, and a non-watermarked image, image, and an integer $n$. Your function should average over the spatial symmetries (recursively to depth $n$) in the watermark to improve the signal-to-noise ratio. Recover the watermark from the watermarked image you created and saved in the last problem.
Figure 1: (a) Example watermark for $n = 0$. (b) For $n = 1$. (c) For $n = 2$.

2 Frequency Domain Filtering

1. Write a function `ideal-lowpass` which takes an integer $n$ and a float `width` as arguments and returns the frequency domain representation of an ideal lowpass filter of size $n \times n$. Your filter should be one for spatial frequencies less than `width` and zero otherwise. Apply your filter to a square image of your choice (of size $n = 2^k$ for integer $k \geq 7$) and `width = 16`. See Figure 2 (a).

2. Write a function `gaussian-lowpass` which takes an integer $n$ and a float `variance` as arguments and returns the frequency domain representation of a Gaussian lowpass filter of size $n \times n$. Your filter should be a Gaussian of variance `variance` centered on the zero spatial frequency. Apply your filter to a square image of your choice (of size $n = 2^k$ for integer $k \geq 7$) and `variance = 32`. See Figure 2 (b).

3. Write a function `ideal-bandpass` which takes an integer $n$ and floats `center` and `width` as arguments and returns the frequency domain representation of an ideal bandpass filter of size $n \times n$. Your filter should be one inside a band of width `width` centered on spatial frequency `center` and zero otherwise. Apply your filter to a square image of your choice (of size $n = 2^k$ for integer $k \geq 7$), `center = 32` and `width = 16`. See Figure 2 (c).

4. Write a function `gaussian-bandpass` which takes an integer $n$ and floats
Figure 2: (a) Ideal lowpass filter. (b) Gaussian lowpass filter. (c) Ideal bandpass filter. (d) Gaussian bandpass filter.

Center and variance as arguments and returns the frequency domain representation of a Gaussian bandpass filter of size $n \times n$. Your filter should be an annulus with Gaussian cross-section with variance $\text{variance}$ and mean $\text{center}$. Apply your filter to a square image of your choice (of size $n = 2^k$ for integer $k \geq 7$) and $\text{variance} = 32$. See Figure 2 (d).

3 Optimal Linear Filtering

There are three directories on the class webpage named signal, noise, and test. These directories contain images of the surface of a silicon wafer imaged with a scanning tunneling microscope by Prof. Sang Han of the UNM Chemical and Nuclear Engineering Department. In the images in the noise and test directories, the atoms which comprise the crystal lattice of the silicon wafer are clearly visi-
ble. Unfortunately, this prominent background lattice complicates the process of automatically classifying the number and size of the island features which are of specific interest to Prof. Han. The images in the signal directory were imaged in a way which suppresses the background lattice but leaves the island features unchanged.

1. Compute the average power of all of the images in the signal directory. Call this image $S(u, v)$. Display $\log S(u, v)$.

2. Compute the average power of all of the images in the noise directory. Call this image $N(u, v)$. Display $\log N(u, v)$.

3. Compute and display the frequency domain representation of the Wiener filter

$$W(u, v) = \frac{S(u, v)}{S(u, v) + N(u, v)}.$$  

4. Apply the Wiener filter to each of the images in the test directory. Display the images before and after Wiener filtering.