CS 362, HW 12

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1. Prove via induction that any graph with maximum degree 3 can be colored with at most 4 colors. Recall that a *coloring* of a graph G is an assignment of a color to each node in G such that the endpoints of each edge in G are assigned different colors. Don't forget to include BC, IH and IS in your proof.
   Hint: Perform induction on, n, the number of nodes in G. In the IS, think about how to make G smaller, so that you can use the IH.

2. The **Subgraph Isomorphism** problem takes as input two undirected graphs G_1 and G_2 and returns TRUE iff G_1 is isomorphic to a subgraph of G_2. Prove that the Subgraph Isomorphism problem is NP-Complete.

3. Show that the next problem is NP-Hard via a reduction from one of the following problems: 3-SAT, VERTEX-COVER, INDEPENDENT-SET, 3-COLORABLE, HAMILTONIAN-CYCLE, or CLIQUE.
   WEIGHTED-ITEM-COVER: You are given (1) a set S of weighted items; (2) a set T of subsets of items; and (3) a number W. You are asked: can you choose a subset S' of items in S with total weight of items in S' no more than W, such that every subset in T contains at least one item in S'? As an example, let $S = \{a, b, c, d\}$, $w(a) = w(b) = w(c) = 1$ and $w(d) = 2$; $T = \{\{a, b, d\}, \{c, d\}, \{b, d\}, \{a, c\}\}$; and $W = 3$. Then the answer is YES since we can set $S' = \{a, d\}$, which has total weight 3 and also ensures that every set in T contains at least one item from S'.

4. Imagine someone gives you a polynomial time algorithm to solve 3-SAT. Describe how you could use this to efficiently find a satisfying assignment for any given 3-CNF formula if that formula is satisfiable.