CS 362, HW 5

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1. Prove that any tree with n nodes has $n-1$ edges. Hint: to get a smaller subproblem, remove a leaf node, i.e. a degree 1 node, from the tree. To prove that any tree has a leaf node, note that trees do not have cycles and consider a path that starts at the root node.

2. A certain product is sold in bags of sizes 3, 5 and 8 kilos. Prove that given an unlimited supply of bags of these sizes, you can fill an order for exactly n kilos for any $n \geq 8$. Hint: Be careful about the base cases, and also the lower bound for your IH.

3. Jesse wants to know the minimum number of bags needed to fill an order of $n \geq 8$ kilos assuming unlimited supply of bags of sizes 3, 5 and 8 kilos. Let $f(n)$ be the minimum number of bags needed.

   (a) Write a recurrence relation for $f(n)$. Hint: Use the minimum function in your recurrence relation; it may also be useful to use the value ∞ for some of your base cases.

   (b) Use your recurrence relation to fill in an array of $f(n)$ values for $n \in [0, 16]$.

   (c) Describe an algorithm to compute $f(n)$ for any given n using your recurrence relation. What is the runtime of your algorithm? How might you augment your algorithm to return the actual set of bags used to achieve the minimum value $f(n)$? Hint: you can put back-arrows in your array.

4. Consider the following function:

```
int f (int n){
  if (n==0) return 2;
  else if (n==1) return 5;
  else{
    int val = 2*f (n-1);
    val = val - f (n-2);
    return val;}}
```

(a) Write a recurrence relation for the *value* returned by f. Solve the recurrence exactly. (Don't forget to check it)

(b) Write a recurrence relation for the *running time* of f. Get a tight asymptotic bound (i.e. Θ) on the solution to this recurrence.