CS 422/522: Digital Image Processing
Homework 1 (Fall '12)

1 Theory

1. The joint p.m.f. of two discrete random variables X and Y is given below:

$$\begin{array}{cc} & Y \\ X & \left[\begin{array}{ccc} 5/12 & 1/18 & 5/72 \\ 1/36 & 1/36 & 5/36 \\ 1/12 & 1/6 & 1/72 \end{array}\right]. \end{array}$$

   Determine whether X and Y are statistically independent.

2. The joint p.m.f. of two discrete random variables X and Y is given below:

$$\begin{array}{cc} & Y \\ X & \left[\begin{array}{ccc} 0 & 1/9 & 1/3 \\ 1/18 & 5/36 & 1/12 \\ 1/36 & 1/4 & 0 \end{array}\right]. \end{array}$$

   Compute and tabulate:

   (a) Marginal p.m.f., $p_X(x_i)$.

   (b) Marginal p.m.f., $p_Y(y_j)$.

3. A p.d.f. for a continuous random variable X is defined as follows:

$$f_X(x) = \begin{cases} 2x/9 & \text{if } 0 < x < 3 \\ 0 & \text{otherwise.} \end{cases}$$

   Find the value of the c.d.f. at 1:

$$F_X(1) = \int_0^1 f_X(x)dx.$$

4. Let X and Y be continuous random variables where $f_X(x) = \frac{1}{\tau}e^{-x/\tau}$ and let $Y = X^2$. Derive an expression for f_Y.

2 Practice

1. Write a function *cumulative-distribution-function* which takes an image as its argument and returns the discrete cumulative distribution function (c.d.f.) for the image:
$$F(j) = \frac{255}{n\,m}\sum_{i=0}^{j} H(i)$$
where n is the number of rows, m is the number of columns, and H is the grey-level histogram. You may assume that the image contains grey-levels in the range $[0,255]$. The c.d.f. should be returned as a *vector.* Compute the discrete c.d.f. for the *frog* image and for an image of your choice. Hint: Although not strictly necessary, learning the Scheme *do* macro might help you.

2. Write a function *histogram-equalize* which takes an image as its argument and returns an image which has been histogram equalized using the discrete c.d.f. as a grey-level transformation. Plot the histograms for the *frog* image and for an image of your choice before and after histogram equalization. You should also show both images before and after histogram equalization. Hint: This is easy to do using *image-map*.

3. Write a function *inverse-cumulative-distribution-function* which takes an image as its argument and returns the discrete inverse cumulative distribution function (discrete i.c.d.f.) for the image. The value of the discrete i.c.d.f. F^{-1} at j is the minimum value k such that $F(k) \geq j$. You may assume that grey-levels are in the range $[0,255]$. The i.c.d.f. should be returned as a Scheme *vector*.

4. Write a function *histogram-match* which takes two images as its arguments and returns an image which is the result of applying the histogram matching grey-level transformation to the first image so that its histogram is matched to that of the second image. Plot the histograms for the *frog* image and for the *cropped-rad* image after its histogram has been matched to that of the *frog* image. Show the transformed *cropped-rad* image. Repeat the above for two equal sized images of your choice. Plot the histograms and show the images before and after (for the second image) histogram matching. Hint: This is easy to do using *image-map*.