Chapter 8 Solutions

8.1 If the upper arm is longer than the lower arm then, as a first approximation, the robot can reach all points within a sphere centered at the joint on the base. However, this answer is only approximate as the lower arm cannot move below the level of the base, so a second approximation is a hemisphere above the base. This approximatoin has to have added to it points below the base that can be reached by the lower arm while the upper arm is parallel to the top of the base, a set of points that depends on the length of the upper arm and the radius of the base. If the lower arm is longer than the upper arm, then there will be an interior sphere radius equal to the difference in the arm lengths that cannot be reached.

8.3 For this simple example, there are three values of the joint angles and three coordinates in space, so there are a couple of simple approaches that can work. One is to usethe result in the previous problem to solve for the joint angles in terms of the given x, y, and z values, using a numerical method. We could then simply move the joint angles from their initial to final positions linearly.

A more interesting possibility is to have the tip of the robot arm trace out a given path, for example, a line segment between the starting and ending positions. If use the parametric form of this line, we can obtain a set of positions along it as the parameter varies from 0 to 1. For each of these intermediate positions, we can numerically solve for the joint angles and have the joint angles move linearly between succesive points. This scheme would give us an approximately linear trajectory.

In general, the problem is far more difficult because a typical robot will have many more degrees of freedom and thus there will be many possible joint angle combinations for a given reachable position. We also have to worry about factosr such as the maximum allowable rate at which joint angles can change.

8.5 There are a number of possibilities. One is to use the turtle representation where the angle is 60 degrees and the distance forward is half the size of the triangle. Then the sequence $FFRRFFRRFRFRRFRRFRFRR$ will draw the edges of the four triangles. Another approach is to have 3 productions, each of which draws one of the interior triangles.

8.17 Let case 0 be the case with no black or no white vertices, case 1 be the case with 1 black or one white vertex, case 2 be the case with two adjacent white vertices and and case 3 be the case with two diagonally opposite white vertices. Coloring consective vertices as 0 or 1, the 16 cases are

0000 0 all white
0001 1 one white
0010 1 one white
0011 2 two consecutive white
0100 1 one white
0101 3 diagonally opposite white
0110 2 two consecutive white
0111 1 one black
1000 1 one white
1001 2 two consecutive white
1010 3 diagonally opposite white
1011 1 one black
1100 2 two consecutive white
1101 1 one black
1110 1 one black
1111 0 all black

8.21 If the values of the function $f(x, y, z)$ are given at the corners of a unit cube with a vertex at the origin and lying in the positive octant, then trilinear interpolation approximates f inside the cube by the function

$$g(x, y, z) = (1-x)(1-y)(1-z)f(0,0,0) + x(1-y)(1-z)f(1,0,0)$$

$$+(1-x)y(1-z)f(0,1,0) + (1-x)(1-y)zf(0,0,1) + xy(1-z)f(1,1,0)$$

$$+x(1-y)zf(1,0,1) + (1-x)yzf(0,1,1) + xyzf(1,1,1).$$

Clearly where the cube is located is unimportant and the values of f can be replaced by samples at the vertices of any parallelepiped.