Chapter 9 Solutions

9.1 $(m+1)^3$

9.3 As u varies over (a, b), $v = \frac{u-a}{b-a}$ varies over $(0, 1)$. Substiuting into the polynomial $p(u) = \sum_{k=0}^{n} c_k u^k$, we have
$q(v) = \sum_{i=0}^{v} d_i v^i = \sum_{k=0}^{n} c_k((b-a)v + a)^k$. We can expand the products on the right and match powers of v to obtain $\{d_i\}$.
9.5 Consider the Berstein polynomial

$$b_{kd}(u) = \left(\begin{array}{c} d \\ k \end{array} \right) u^k (1-u)^{d-k}.$$

For $k = 0$ or $k = d$, the maximum value of 1 is at one end of the interval (0,1) and the minum is at the other because all the zeros are at 1 or 0. For other values of k, the polynomial is 0 at both ends of the interval and we can differentiate to find that the maximum is at $u = k/d$. Substituting into the polynomial, the maximum value is $\frac{d!}{d^d} \frac{k^k}{k!} \frac{(d-k)^{d-k}}{(d-k)!}$ which is always between 0 and 1.

9.6 For the cubic B–spline, the blending functions are

$$\mathbf{b}(u) = \frac{1}{6} \left[\begin{array}{c} (1-u)^3 \\ 4 - 6u^2 + 3u^3 \\ 1 + 3u + 3u^2 - 3u^3 \\ u^3 \end{array} \right],$$

and the first two deriviatives are

$$\mathbf{b}'(u) = \frac{1}{2} \left[\begin{array}{c} -(1-u)^2 \\ -4u + 3u^2 \\ 1 + 2u - 3u^2 \\ u^2 \end{array} \right],$$

$$\mathbf{b}''(u) = \left[\begin{array}{c} 1-u \\ -4 + 6u \\ 1 + 3u + 3u^2 - 3u^3 \\ u^3 \end{array} \right].$$

Consider the B–spline $p(u)$ determined by the control points P_{i-2}, P_{i-1}, P_i, and P_{i+1} and next B–spline determined by P_{i-1}, P_i, P_{i+1}, and P_{i+2}. Thus,

$$p(u) = \left[\begin{array}{cccc} P_{i-2} & P_{i-1} & P_i & P_{i+1} \end{array} \right] \mathbf{b}(u)$$

$$q(u) = \begin{bmatrix} P_{i-1} & P_i & P_{i+1} & P_{i+2} \end{bmatrix} \mathbf{b}(u)$$

The values of $p(u)$ and its first two derivatives at $u = 1$ have to be compared with the values of $q(u)$ and its first two derivatives ar $u = 0$. Using the above equations, we find

$$p(1) = q(0) = \frac{1}{6}(P_{i-1} + 4P_i + P_{i+1}),$$

$$p'(1) = q'(0) = \frac{1}{2}(P_{i+1} - P_{i-1}),$$

$$p''(1) = q''(0) = P_{i-1} + 2P_1 + 2P_{i+1}.$$

These results are sufficient to verify C^2 continuity because all the derivatives of polynomials are continuous except possibly at the join points.

9.7 Proceeding as on page 374, we have the interpolating control point array $\mathbf{q}$ and can form the interpolating polynomial

$$p(u) = \mathbf{u}^T \mathbf{M}_I \mathbf{q},$$

where $\mathbf{M}_I$ is the interpolating geometry matrix. This polynomial can also be written as a Bezier polynomial

$$p(u) = \mathbf{u}^T \mathbf{M}_B \mathbf{p},$$

for properly chosen control points $\mathbf{p}$. If these representations are to yield the same polynomial, we must have

$$\mathbf{p} = \mathbf{M}_B^{-1} \mathbf{M}_I \mathbf{q}.$$

Thus, we find the correct control points to convert the interpolating polynomial to an equivalent Bezier polynomial and then use our abiliuty to render Bezier polynomials efficiently.

9.9 If we have two patches that share an edge and subdivide only the patch on one side of this edge, we can create a crack. The middle shared endpoint on the subdivided patch does not have to lie on original edge. We can either create an extra triangle from the original endpoints and this middle point to fill the crack or we can triangulate the unsubdivided patch to meet the new subdivided edge, i.e. we rplace the unsubdivided patch by a set of triangles that use three of the edges of the patch and the subdivided edge.

9.11 Although the curves are continuous, when we have only G^1 continuity, there is a discontinuity in the velocity at which we trace the curve. In an animation we might see changes in velocity as objects move along paths described with only G^1 continuity.

9.13 One simple test is to use the *twist* (page 361). Suppose the four corners of the patch are given by $\mathbf{p}_{00}$, $\mathbf{p}_{01}$, $\mathbf{p}_{10}$, and $\mathbf{p}_{11}$. These points form a quadrilateral that will be flat if $\mathbf{p}_{00} - \mathbf{p}_{01} + \mathbf{p}_{10} - \mathbf{p}_{11}$ is zero. A simple test of flatness is to measure the magnitude of this term.

9.15 For $r = 0$ we get the line between P_0 and P_2. For $r = \frac{1}{2}$ we get the parabola $u^2 P_0 + 2u(1-u)P_1 + (1-u)^2 P_2$ which passes through P_0 and P_2. For $r > \frac{1}{2}$, we obtain hyperbolas, and for $r < \frac{1}{2}$, we obtain ellipses. Thus, we can use NURBSs to obtain both parametric polynomial curves and surfaces, and to obtain quadric surfaces.

9.17 We can write the Hermite surface as

$$\mathbf{p}(u, v) = \mathbf{u}^T \mathbf{M}_H \mathbf{Q} \mathbf{M}_H^T \mathbf{v} = \mathbf{u}^T \mathbf{A} \mathbf{v},$$

where $\mathbf{Q}$ contains the control point data and $\mathbf{M}_H$ is the Hermite gfeometry matrix. If evaluate $\mathbf{p}$, $\frac{\partial \mathbf{p}}{\partial u}$, $\frac{\partial \mathbf{p}}{\partial v}$, and $\frac{\partial^2 \mathbf{p}}{\partial u \partial v}$ at the corners we find that the 16 values in the matrix $\mathbf{A}$ are the 4 values at the 4 corners of the patch, the first partial derivatives $\frac{\partial \mathbf{p}}{\partial v}$ and $\frac{\partial \mathbf{p}}{\partial u}$ at the corners and the first mixed partial derivative $\frac{\partial^2 \mathbf{p}}{\partial u \partial v}$ at the corners