Chapter 8 Solutions

8.17 Let case 0 be the case with no black or no white vertices, case 1 be the case with 1 black or one white vertex, case 2 be the case with two adjacent white vertices and and case 3 be the case with two diagonally opposite white vertices. Coloring consective vertices as 0 or 1, the 16 cases are

0000 0 all white
0001 1 one white
0010 1 one white
0011 2 two consecutive white
0100 1 one white
0101 3 diagonally opposite white
0110 2 two consecutive white
0111 1 one black
1000 1 one white
1001 2 two consecutive white
1010 3 diagonally opposite white
1011 1 one black
1100 2 two consecutive white
1101 1 one black
1110 1 one black
1111 0 all black

12.5 Using the case numbers from page 336, for each case we use symmetry arguments. For example, for case 1, the black vertex can be any of the 8 vertices and we can swtich white and black, yielding 16 possibilities. For case 6, the adjacent blacks can be along any of the 12 edges but there are 2 places we can place the third black and not have all three blacks on the same face, which yields 24 cases, but then we can switch black and white, yielding 48 possibilities. The numbers of each case are

```
0    2
1   16
2   24
3   24
4    8
5   48
6   48
7   16
8    6
9    8
10   6
11  24
12  24
13   2
```

12.9 If the values of the function $f(x, y, z)$ are given at the corners of a unit cube with a vertex at the origin and lying in the positive octant, then trilinear interpolation approximates f inside the cube by the function

$$g(x, y, z) = (1-x)(1-y)(1-z)f(0,0,0) + x(1-y)(1-z)f(1,0,0)$$

$$+(1-x)y(1-z)f(0,1,0) + (1-x)(1-y)zf(0,0,1) + xy(1-z)f(1,1,0)$$

$$+x(1-y)zf(1,0,1) + (1-x)yzf(0,1,1) + xyzf(1,1,1).$$

Clearly where the cube is located is unimportant and the values of f can be replaced by samples at the vertices of any parallelepiped.