Iterated Maps and Measures of Uncertainty

CS 523: Complex Adaptive Systems
Assignment 1: Due: Aug. 31, 2015

1 Introduction

This is a warm-up exercise to get you thinking about the mysteries of complex systems, to gain experience with the challenges of measuring a system's complexity, let you familiarize yourself with the formatting requirements for the assignments, and to make sure that you know how to generate plots that are labeled correctly.

The assignment is due at the beginning of class on Aug. 31. Please hand in a printed copy of a 2-3 page paper describing your assignment, formatted according to the ACM SIG Proceedings format, available from:
`http://www.acm.org/sigs/publications/proceedings-templates`,
together with the output, and the code you wrote to complete the assignment. I prefer to receive hard copy, but if you submit your assignment electronically, please send the paper in .pdf format rather than .docx or .tex.

2 Iterated Map

Consider the following two-dimensional map:

$$x_{t+1} = a - x_t^2 + by_t$$

$$y_{t+1} = x_t$$

where a and b are adjustable parameters. An orbit (also called a trajectory) of the system consists of a starting point (x_0, y_0) and its iterated values. Similar to the logistic map, these dynamics depend dramatically on the choice of the constants a and b, and the starting point (x_0, y_0). Let's explore the behavior of these equations.

2.1 Exploration of the dynamics

First off, implement the equations in the programming language of your choice, and make some simple time-series plots, with time (number of iterations) on the horizontal axis and x_t and y_t (plotted in two different colors and labeled with a legend) on the vertical axis, and then some two-dimensional plots of x_t vs. y_t. Experiment with different values for a, b, x_0, and y_0. Now, select three informative plots, e.g., plots that illustrate different dynamical regimes (fixed point, limit cycle, more complex behavior), and report them as Figure 1 of your paper. For the more complex dynamics, iterate the map for at least 5000 time steps. Make sure to label the axes clearly and include a figure caption that gives the two parameter values and the initial conditions for each plot. Your plots for this figure must use DIFFERENT parameter values from those specified in the remaining parts of the assignment and from your fellow students.

2.2 Sensitivity to initial conditions

Next, let's study how sensitive the map is to small changes in initial conditions. Using the parameter values $a = 1.29$, $b = 0.3$, and an initial condition chosen randomly between 0 and 1 (random means that no two students' plots should look identical), experiment with small variations in the initial condition, plotting a time series of one variable (say x) for two runs. Select two example runs and report them as Figure 2, panel a. Next, in Figure 2, panel b, make a plot of how long it takes two orbits to diverge as a function of the difference in their initial condition. Explain how you define divergence.

2.3 Bifurcations

Finally, make a bifurcation plot, similar to that shown in Flake, Figure 10.7 but for the map we have been studying. For this plot, vary a from 0.5 to 1.4, but set b to 0.3. Choose your own initial conditions as before (between 0 and 1) and use the same initial condition for each different value of a that you plot. Experiment to find a step size in a that is small enough to reveal the interesting structure and a number of iterations that reveals the size of the orbit for each value of a. Report this plot as panel a of Figure 3, and then zoom in on a small region of the plot, so we can see more detail, and report that as panel b. Remember to write a figure caption that reports the initial condition you used, the step size, and any other pertinent information about the figure. If you fail to find an interesting pattern, consider trying again with a different initial condition.

3 Entropy Calculation

For each value of a used in the bifurcation plot, calculate the steady state entropy of x in the map. That is, after the map reaches the steady state, for each unique value of x_i calculate:

$$H_a(x) = -\sum_{x_i \in x} Pr[x_i] \log Pr[x_i] \tag{1}$$

Be sure to report how you define steady state. Plot as panel c of Figure 3, H vs a (that is, a on the horizontal axis and H_a on the vertical axis). Discuss how the entropy changes as a increases, and how changes in entropy are related to patterns you observed in the bifurcation plot.

4 Extra Credit

Consider a new model based on the map above. In this model, before the map is iterated, a is chosen uniformly at random to be one of two different values 0.75 or 1.25. What is the conditional entropy of the steady state value of x given a ? What is the mutual information between a and x? Be sure to state the equations that you are using for conditional entropy and mutual information, and cite your source.

5 What to hand in

Hand in a short report (not more than three pages) that describes your implementation. Think carefully about which runs to report and select those that best illustrate the phenomena you want to show. Document the programming language, any external or built-in libraries that you used, major design decisions, etc.

For each assigned experiment, include a short discussion of your results and what you think they mean. I am not interested in looking at endless printouts of your dynamical system, so think carefully about how to present your results in a convincing but succinct manner. Organize your paper into the following sections:

- Introduction: Summarize briefly the problem you are trying to solve and how you went about it.
- Exploring Dynamics
- Sensitivity to Initial Conditions
- Bifurcation plots
- Entropy
- Discussion and Conclusion: Summarize any interesting discoveries you made. If you did not achieve the expected results, explain why you think that happened and how you might address it in future work.
- Bibliography

Remember to cite all of your sources using a consistent citation style and include a proper bibliography.

Please include a listing of your code and instructions for how to run it as an appendix to your report.

6 Late Policy

You are allowed three free "late days" to be used at your discretion throughout the semester. After you have used up your late days, I will deduct 10% per day from the grade you would have received on any late work. Since the projects will become more involved throughout the semester, I strongly recommend that you save your late days for when you really need them.