Retrograde Analysis

R.A. has been used for many famous examples.

- Recent proof that Rubik's cube can be solved in 20 moves or fewer.
- Solution of Oware game in 2002 (900 billion positions considered).
- Solution of Checkers in 2007.
- Some chess endgames, e.g. KQR vs QR.

Announcement

Your final assignment (#5). Due Thurs, 12/5

10 points: problem 4.18.

50 points: Write code to do retrograde analysis on the game of Sprouts. It should be able to classify positions as wins or losses for the player to move.

You may do this assignment in teams of two people.

Rules of Sprouts

Start out with k vertices and no edges.

One move: create a new vertex, and draw 2 edges from it to existing vertices (these can be the same).

Rules: No vertex degree can be > 3

Edges must be drawn without crossing.

Whoever makes the last move wins!

Representing Sprouts

What do we need to know about a sprouts board?

The vertices.

The edges.

Representing Sprouts

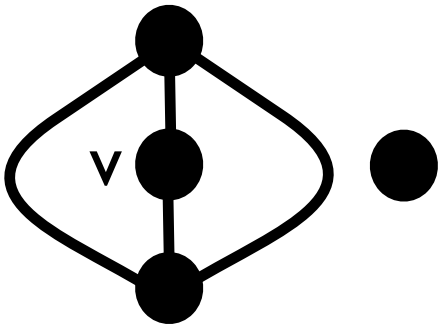
What do we need to know about a sprouts board?

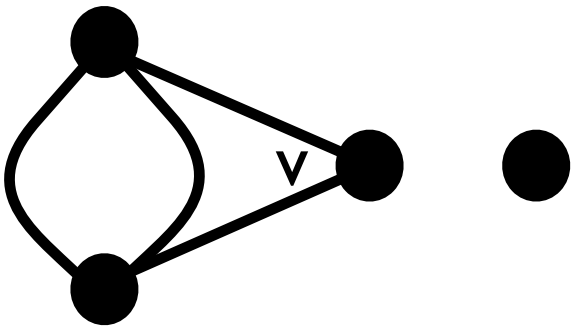
The vertices.

The edges.

and how they are drawn in the plane!

Two different positions





Note: as a graph, only the drawing has changed. The vertices and edges remain the same.

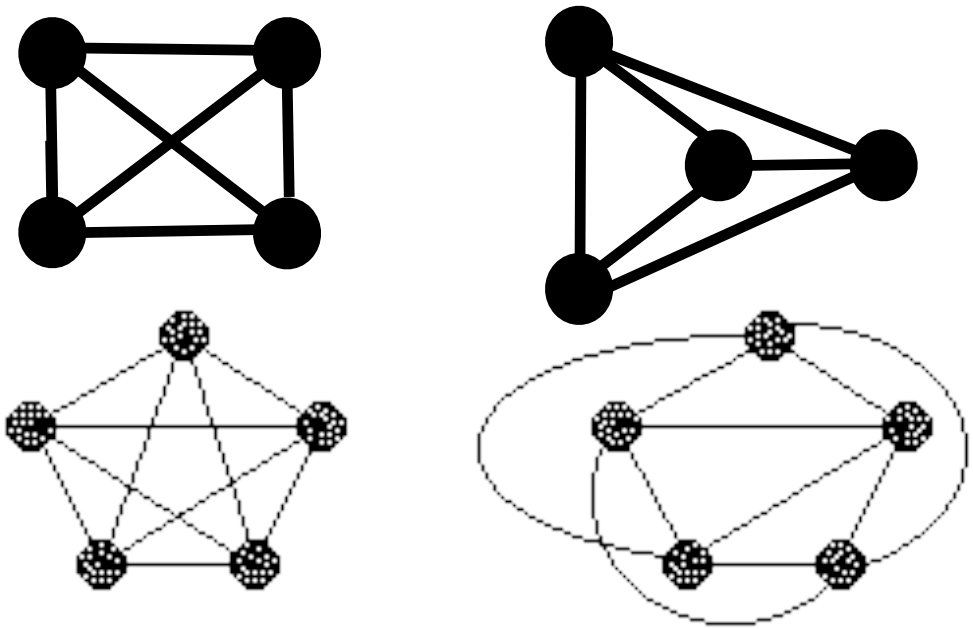
(Also note the duplicate edge.)

Left one is a loss. Right one is a win.

Planar graphs

We say a graph (V, E) is planar if it can be drawn on a sheet of paper without any crossings.

Example: a 4-clique is planar. A 5-clique is not.



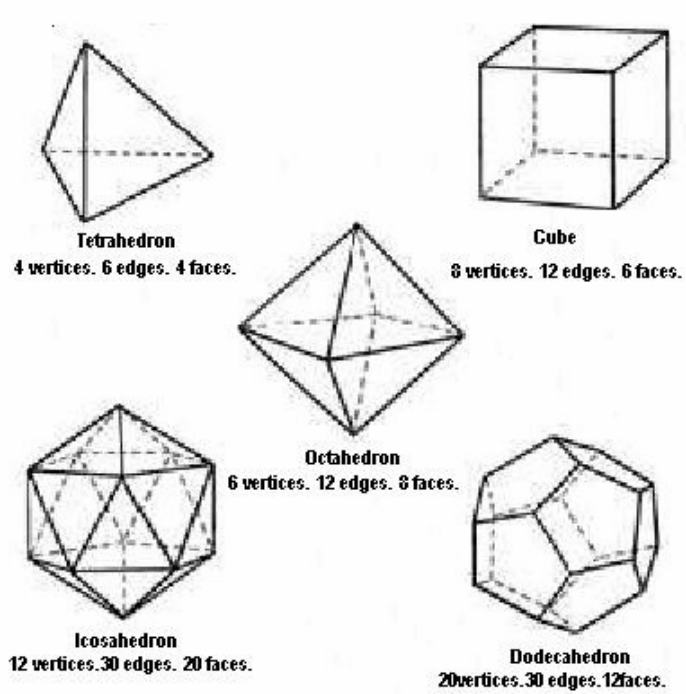
Cool planarity facts

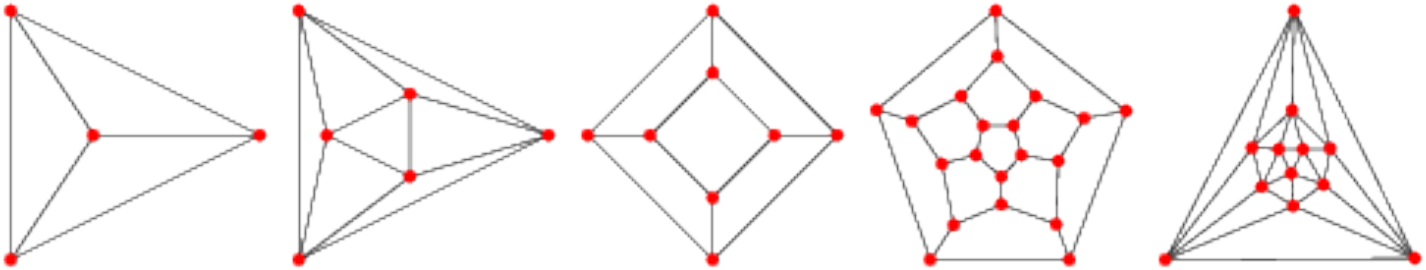
A planar graph can always be drawn in the plane using straight-line segments for the edges. This can be done in $O(n + m)$ time.

Koenig's Theorem: A graph is planar if and only if it has no 5-clique or utility graph minor.

4-color Theorem (Appel-Haken): The vertices of a planar graph can be properly colored with only 4 colors. (no monochrome edge). First "computer proof"

Platonic Solids





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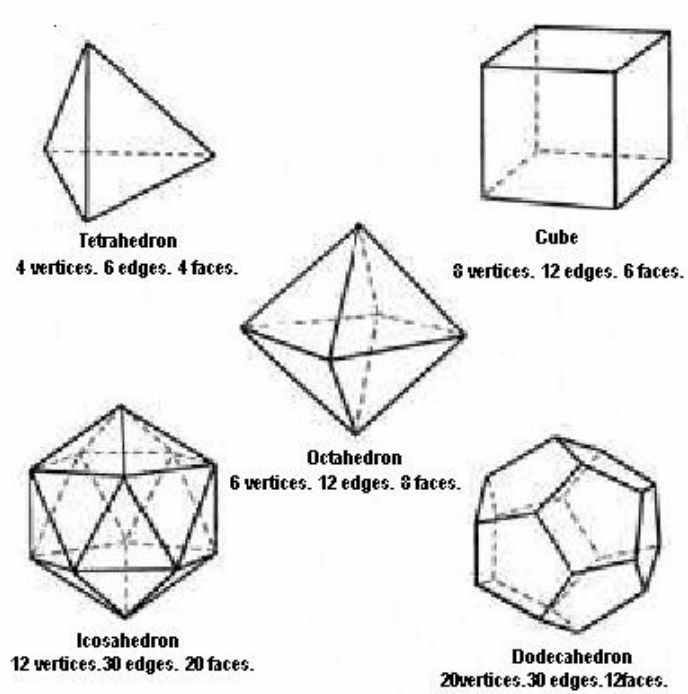


Euler's Formula:

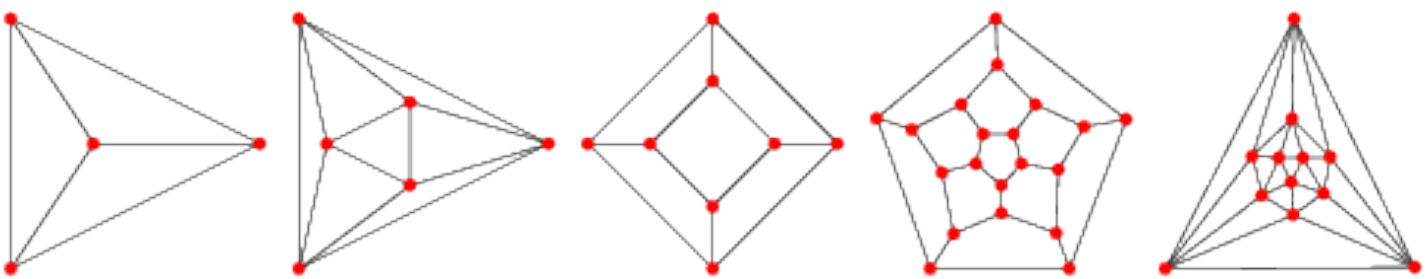
$v - e + f = 2$

v: # vertices

e: # edges

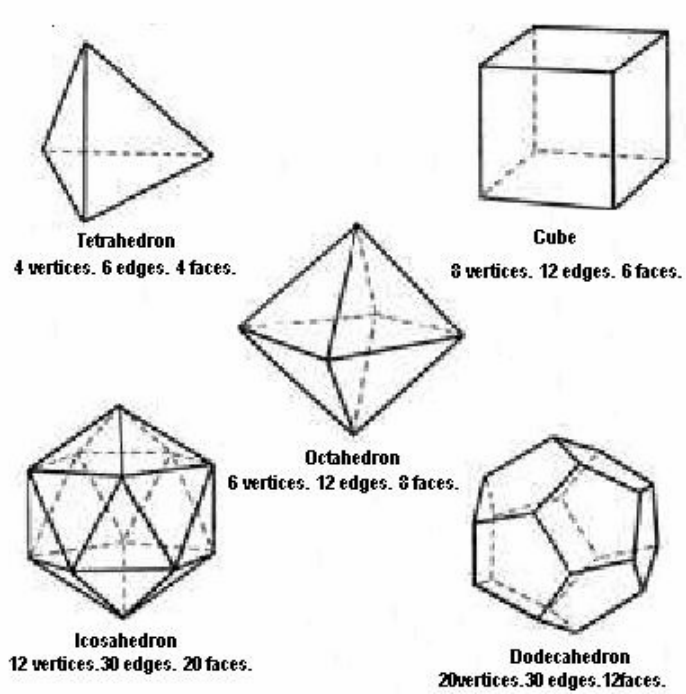
f: # faces

Applies to any connected plane graph



NB: Don't forget the outer face!

Platonic Solids



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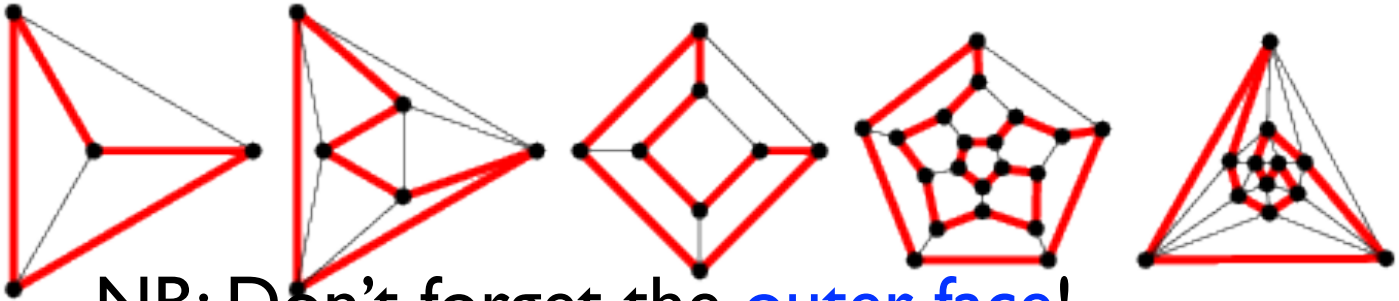

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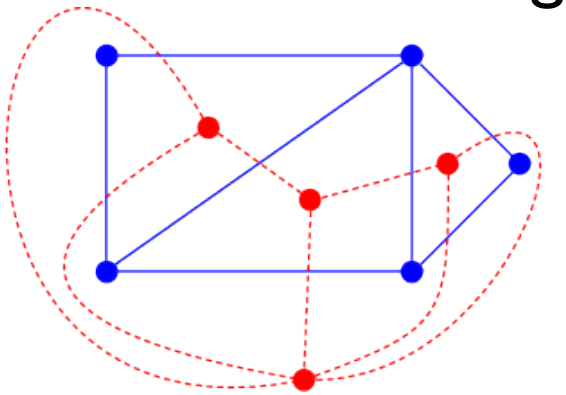
Duality

"Flip" the role of faces and vertices to get the platonic solids back again.

Cube and octahedron: duals

Any planar graph can be "dualized" by putting a vertex in each face, and making an edge cross each old edge.



Sprouts Moves

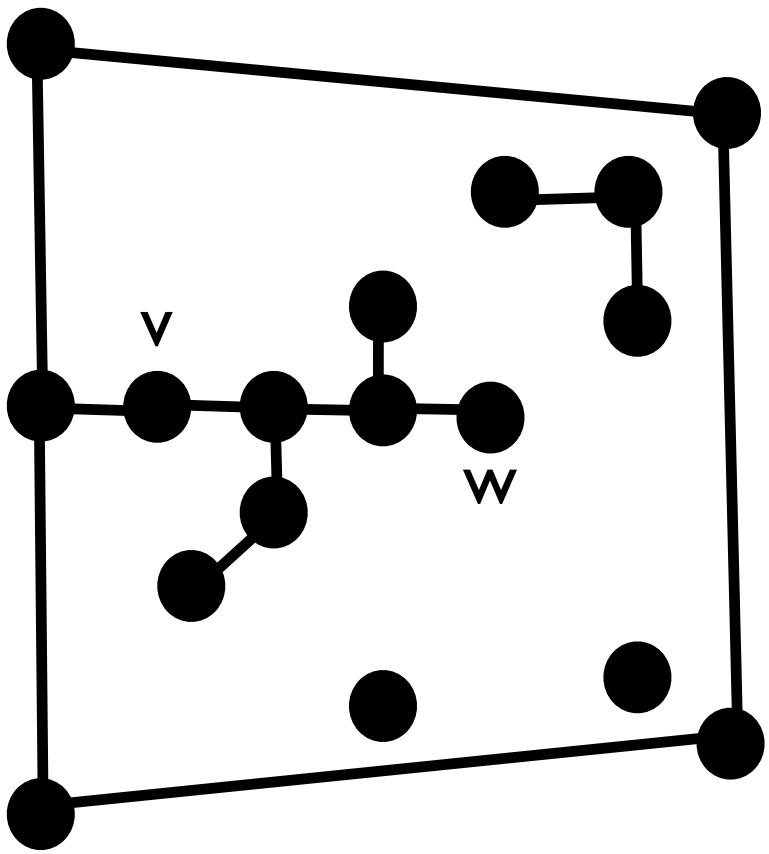
A sprouts move either

* joins 2 components, or

* splits a face into two parts

In the former case, it doesn't really matter how. In the latter case, there is a question of how to divide up the other stuff in the face.

Example



How many ways can we join v to w?

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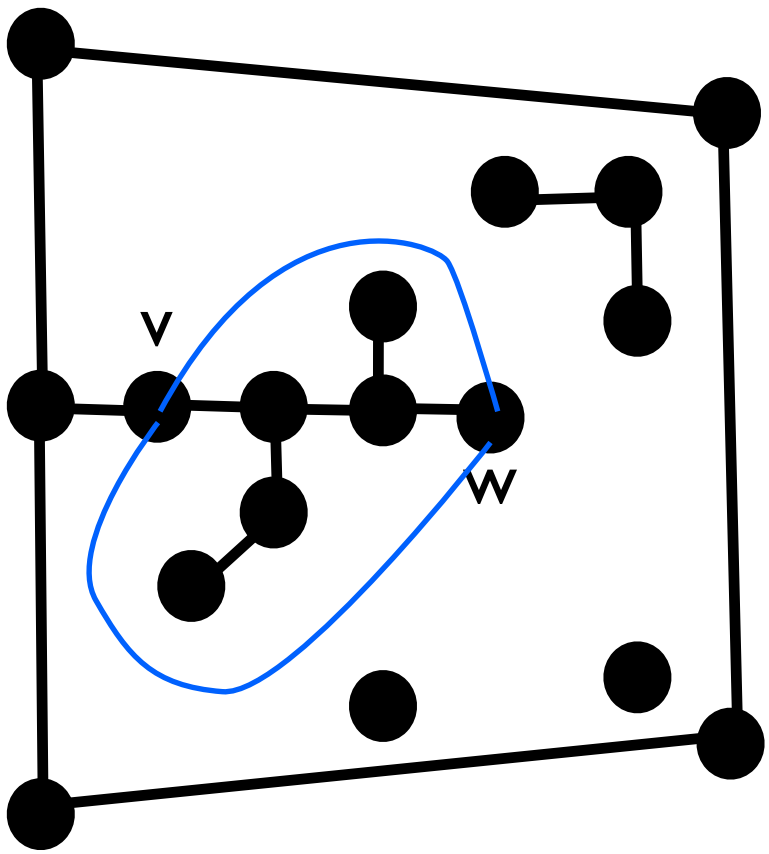

How many ways can we join v to w?

2 main directions.

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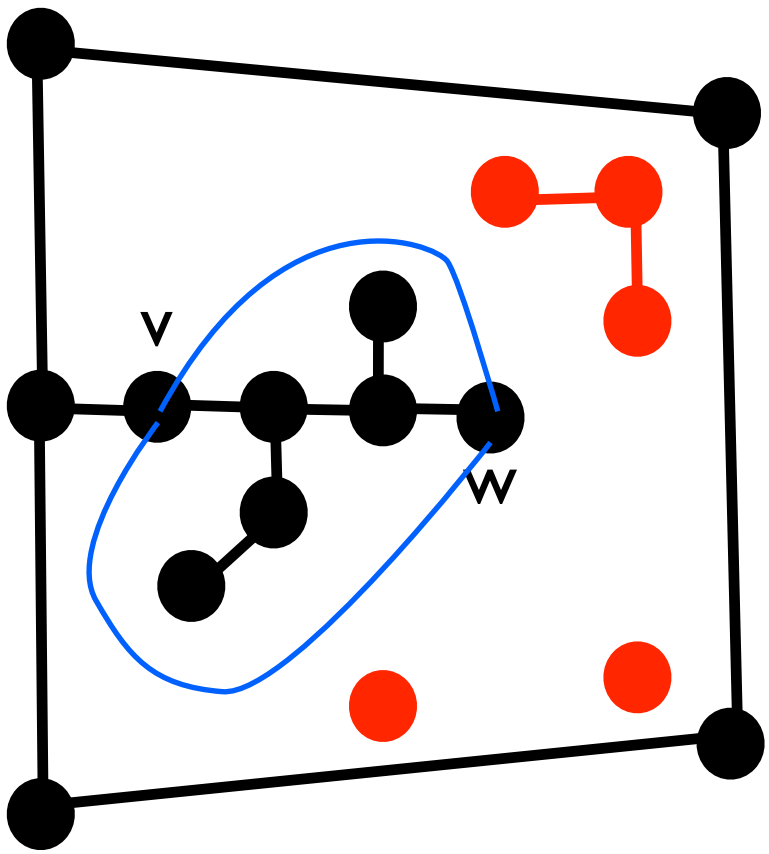
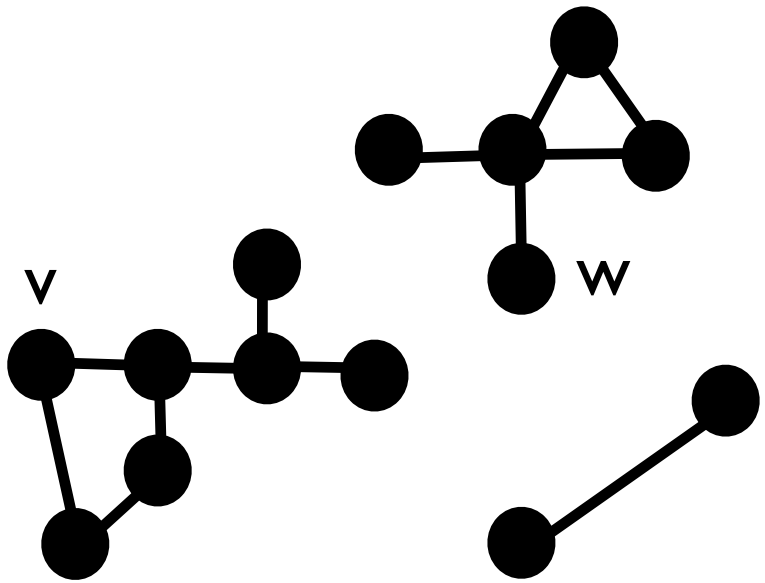


How many ways can we join v to w?

2 main directions. 2^3 choices for which "blobs" to enclose, so: 16 in all.

Example 2



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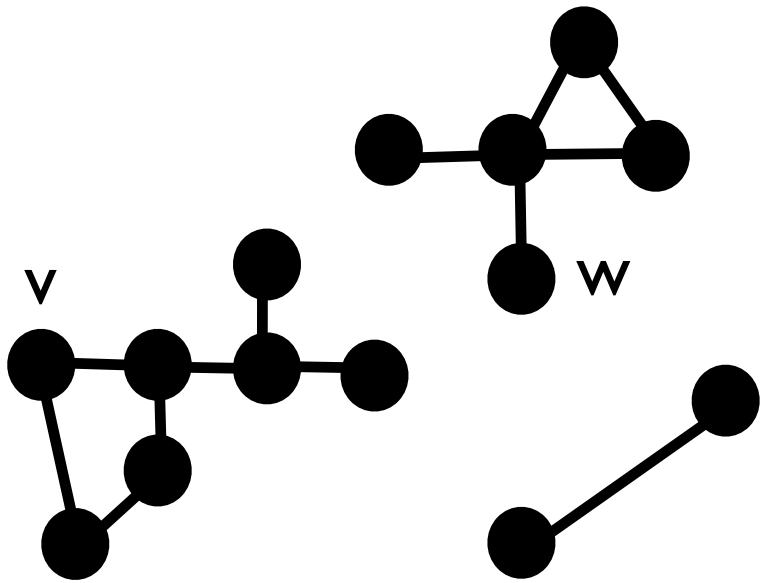


How many ways can we join v to w?

A sprouts move

(1) Which region does the edge lie inside?

(2) Which boundaries are its endpoints on?

(3) Indices for the vertices within the boundaries?

(4) If edge splits a region, which of the other boundaries go “inside”? Which go “outside”?

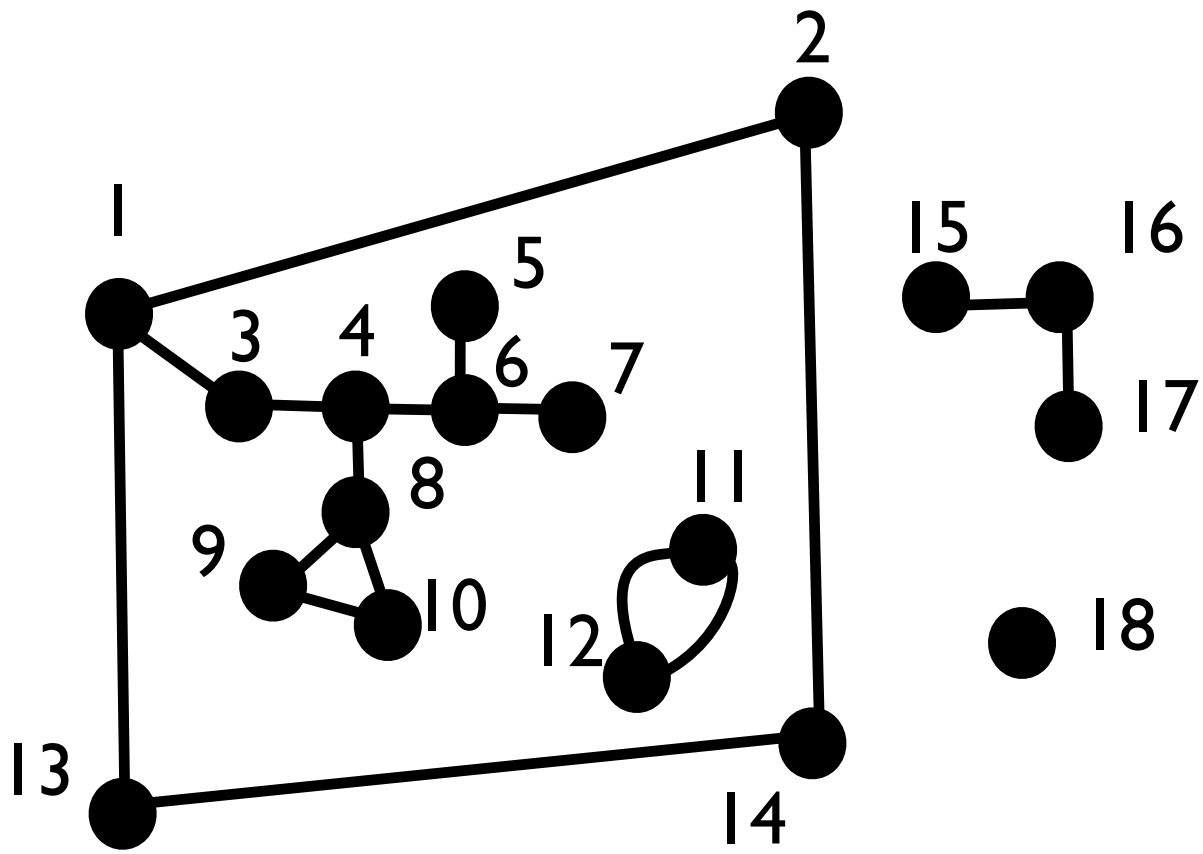
Keep track of:

Regions (faces): areas of the plane, separated by edges.

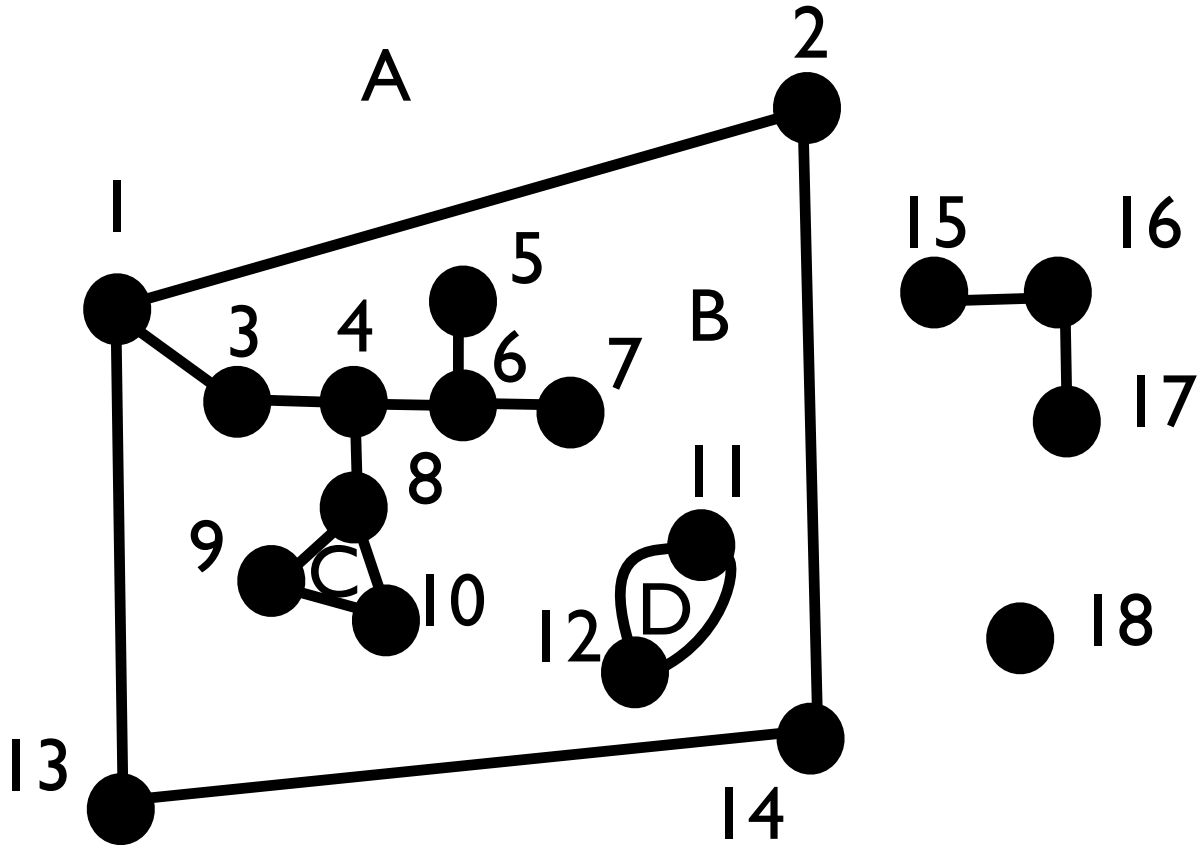
Boundaries: vertices touching a region, joined by edges. Careful: vertices can be repeated!

Vertices: Need to know their degrees (how many edges touch them). Careful: edges can repeat!

An example position

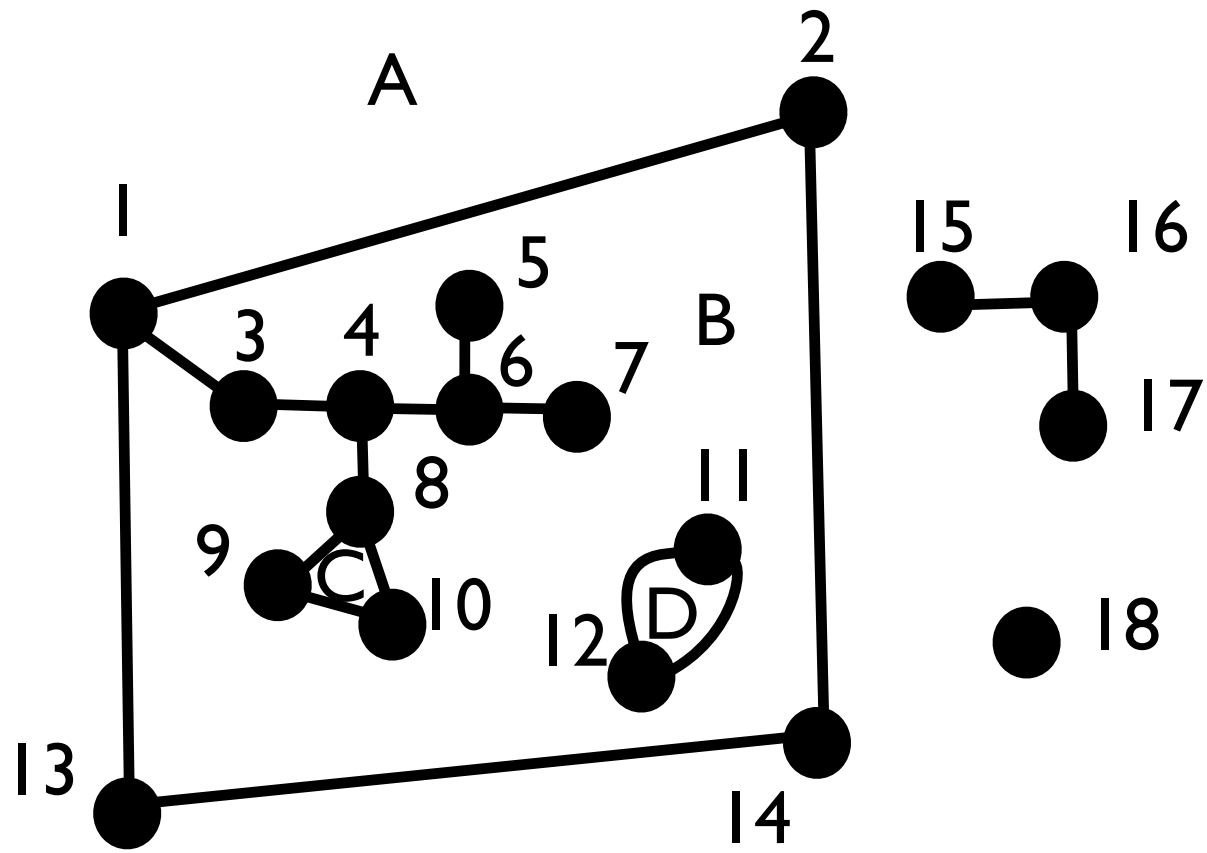


Regions: A, B, C, D



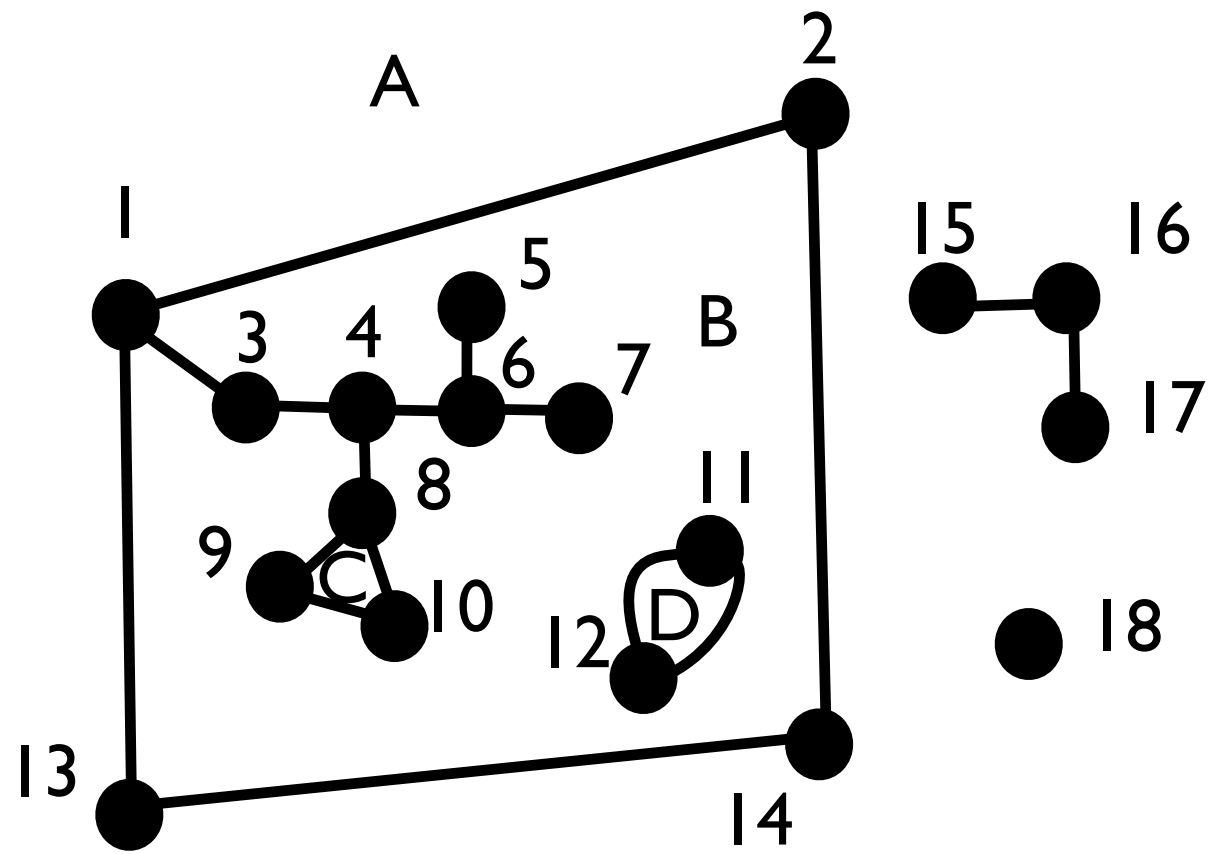
Regions: A, B, C, D

Region A has 3 boundaries:
(1,2,14,13) and (15, 16, 17, 16) and 18

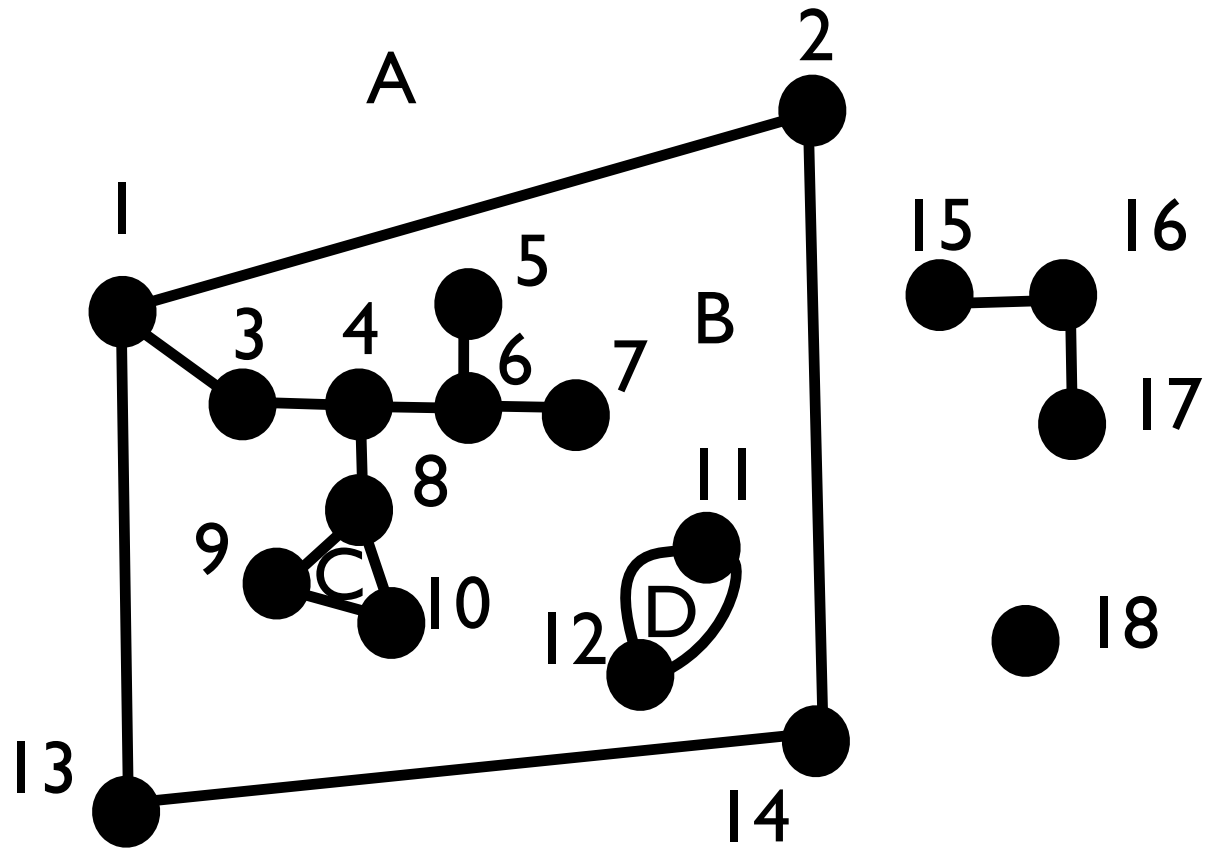


Regions: A, B, C, D

Region C has one boundary: (8, 10, 9)
Region D has one boundary: (11, 12)



Region B has 2 boundaries:
(1, 3, 4, 6, 5, 6, 7, 6, 4, 8, 10, 9, 8, 4, 3, 1, 13, 14, 2)
and(11, 12)



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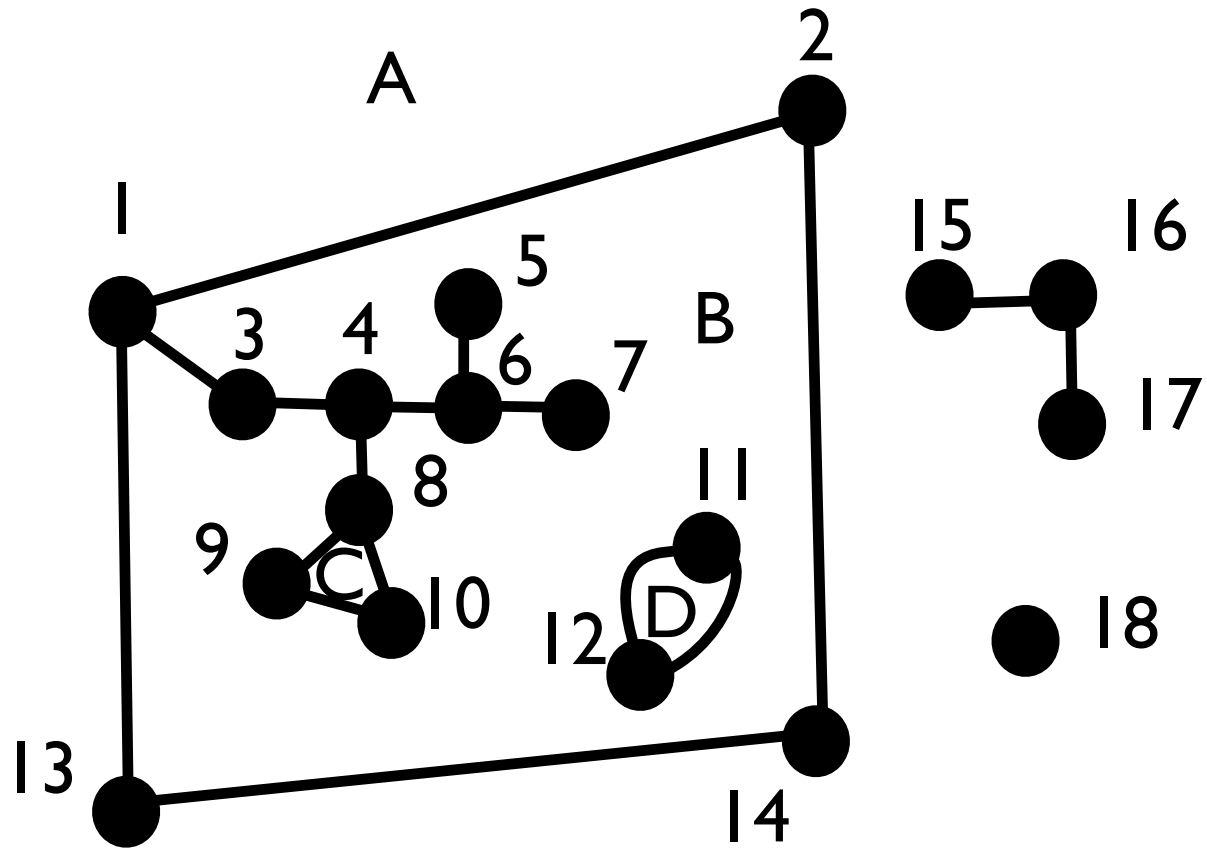

Isomorphism

Sometimes we can figure out that two or more positions are “essentially the same”.

In this case, we can merge them in our game digraph, since they will get the same label (win/loss) anyway.

Cleverly doing this can massively shrink the game digraph

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Is one position just a relabelling of another?

Up to irrelevant details?