## AXDInterpolator

## A Tool for Computing Interpolants for Arrays with MaxDiff

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## Motivation

- The theory of arrays with extensionality does not have quantifier-free interpolation [5]


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- Interpolant in $\langle=, w r, r d\rangle$ is

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\begin{aligned}
& \exists j \cdot(\operatorname{rd}(h, j) \neq \\
& \operatorname{rd}\left(h^{\prime}, j\right) \wedge \forall k .(k \neq j \rightarrow \\
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\end{aligned}
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- Interpolant in $\langle=, \mathrm{wr}, \mathrm{rd}, \operatorname{diff}\rangle$ is let $j=$ $\operatorname{diff}\left(h, h^{\prime}\right)$ be in $h^{\prime}=$ $\operatorname{wr}(h, j, r d(h, j))$


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- The diff operator in this work returns 0 if the two input arrays are the same and otherwise returns the biggest index where the input arrays are different
- Endowing this semantic on diff allows the theory to formalize desirable specifications (in particular, the length function) without quantifiers of bounded arrays


## Contributions

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- Provided support for the quantifier-free fragment of the index theories $\mathcal{T O}, \mathcal{I D} \mathcal{L}$, and $\mathcal{L I} \mathcal{A}$.
- Designed an architecture allowing the system to use different interpolation engines as black boxes. Currently, we support iZ3, SMTInterpol, and MathSAT.


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\forall y, i, e . & i \geq 0 \rightarrow \operatorname{rd}(\operatorname{wr}(y, i, e), i)=e \\
\forall y, i, j, e . & i \neq j \rightarrow \operatorname{rd}(\operatorname{wr}(y, i, e), j)=\operatorname{rd}(y, j) \\
\forall x, y . & x \neq y \rightarrow \operatorname{rd}(x, \operatorname{diff}(x, y)) \neq \operatorname{rd}(y, \operatorname{diff}(x, y)) \\
\forall x, y, i . & i>\operatorname{diff}(x, y) \rightarrow \operatorname{rd}(x, i)=\operatorname{rd}(y, i) \\
\forall x . & \operatorname{diff}(x, x)=0 \\
\forall x . i & i<0 \rightarrow \operatorname{rd}(x, i)=\perp \\
\forall i . & \operatorname{rd}(\varepsilon, i)=\perp \tag{7}
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\forall x, y . & x \neq y \rightarrow \operatorname{rd}(x, \operatorname{diff}(x, y)) \neq \operatorname{rd}(y, \operatorname{diff}(x, y)) \\
\forall x, y, i . & i>\operatorname{diff}(x, y) \rightarrow \operatorname{rd}(x, i)=\operatorname{rd}(y, i) \\
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\end{array}
$$

- As an effect of the above axioms, we have that an array $x$ is undefined outside the interval $[0,|x|]$, where $|x|$ is defined as $|x|:=\operatorname{diff}(x, \varepsilon)$.


## Theory Arrays with MaxDiff (Cont'd)

Given array variables $a, b$, we define by mutual recursion the sequence of array terms $b_{1}, b_{2}, \ldots$ and of index terms $\operatorname{diff}_{1}(a, b), \operatorname{diff}_{2}(a, b), \ldots$ :

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b_{1}:=b
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$$
\operatorname{diff}_{1}(a, b):=\operatorname{diff}\left(a, b_{1}\right)
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$$
\begin{array}{cl}
b_{1}:=b ; & \operatorname{diff}_{1}(a, b):=\operatorname{diff}\left(a, b_{1}\right) ; \\
b_{k+1}:=\operatorname{wr}\left(b_{k}, \operatorname{diff}_{k}(a, b), \operatorname{rd}\left(a, \operatorname{diff}_{k}(a, b)\right)\right) ; & \operatorname{diff}_{k+1}(a, b):=\operatorname{diff}\left(a, b_{k+1}\right)
\end{array}
$$

## Theory Arrays with MaxDiff - Lemma

The conjunctive formula

$$
\begin{equation*}
\operatorname{diff}_{1}(a, b)=k_{1} \wedge \cdots \cdots \wedge \operatorname{diff}_{l}(a, b)=k_{l} \tag{8}
\end{equation*}
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is equivalent modulo $\mathcal{A R} \mathcal{D}$ to the conjunction of the following five formulæ:

$$
\begin{gather*}
k_{1} \geq k_{2} \wedge \cdots \wedge k_{I-1} \geq k_{l} \wedge k_{l} \geq 0  \tag{9}\\
\bigwedge_{j<1}\left(k_{j}>k_{j+1} \rightarrow \operatorname{rd}\left(a, k_{j}\right) \neq \operatorname{rd}\left(b, k_{j}\right)\right)  \tag{10}\\
\bigwedge_{j<1}\left(k_{j}=k_{j+1} \rightarrow k_{j}=0\right)  \tag{11}\\
\bigwedge_{j \leq I}\left(\operatorname{rd}\left(a, k_{j}\right)=\operatorname{rd}\left(b, k_{j}\right) \rightarrow k_{j}=0\right)  \tag{12}\\
\forall h\left(h>k_{I} \rightarrow \operatorname{rd}(a, h)=\operatorname{rd}(b, h) \vee h=k_{1} \vee \cdots \vee h=k_{l-1}\right)
\end{gather*}
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## Separated Pairs

A pair of sets of quantifier-free formulae $\Phi=\left(\Phi_{1}, \Phi_{2}\right)$ is a separated pair iff
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(1) $\Phi_{1}$ contains equalities of the form $\operatorname{diff}_{k}(a, b)=i$ and $a=\mathrm{wr}(b, i, e)$; moreover if it contains the equality $\operatorname{diff}_{k}(a, b)=i$, it must also contain an equality of the form $\operatorname{diff}_{l}(a, b)=j$ for every $l<k$;

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(2) $\Phi_{2}$ contains Boolean combinations of $T_{1}$-atoms and of atoms of the forms: $\left\{r d(a, i)=r d(b, j), \quad \operatorname{rd}(a, i)=e, \quad e_{1}=e_{2}\right\}$, where $a, b, i, j, e, e_{1}, e_{2}$ are variables or constants of the appropriate sorts.

## M-Instantiations

If $\mathcal{I}$ is a set of $\mathcal{T}_{\mathcal{I}}$-terms, an $\mathcal{I}$-instance of a universal formula of the kind $\forall i \phi$ is a formula of the kind $\phi(t / i)$ for some $t \in \mathcal{I}$.

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Let $\mathcal{I}$ be a set of $\mathcal{T}_{\mathcal{I}}$-terms and let $\Phi=\left(\Phi_{1}, \Phi_{2}\right)$ be a separated pair; we let $\Phi(\mathcal{I})=\left(\Phi_{1}(\mathcal{I}), \Phi_{2}(\mathcal{I})\right)$ be the smallest separated pair satisfying the following conditions:

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- if $\Phi_{1}$ contains $a=\mathrm{wr}(b, i, e)$, then $\Phi_{2}(\mathcal{I})$ contains all the $\mathcal{I}$-instances of the equivalent formula $(i \geq 0 \rightarrow \operatorname{rd}(a, i)=e) \wedge \forall h(h \neq i \rightarrow \operatorname{rd}(a, h)=\operatorname{rd}(b, h))$;


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- if $\Phi_{1}$ contains the conjunction $\bigwedge_{i=1}^{\prime} \operatorname{diff} f_{i}(a, b)=k_{i}$, then $\Phi_{2}(\mathcal{I})$ contains formulae (9), (10), (11), (12) as well as all I-instances of the formula (13).


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The full instantiation of $\Phi=\left(\Phi_{1}, \Phi_{2}\right)$ is the separated pair $\Phi\left(\mathcal{I}_{\Phi}^{\infty}\right)=\left(\Phi_{1}\left(\mathcal{I}_{\Phi}^{\infty}\right), \Phi_{2}\left(\mathcal{I}_{\Phi}^{\infty}\right)\right)$ (which is usually not finite). A separated pair $\Phi=\left(\Phi_{1}, \Phi_{2}\right)$ is $M$-instantiated iff $\Phi=\Phi\left(\mathcal{I}_{\Phi}^{M}\right)$

## M-Instantiations - Pseudo Code for quantifier-free $\mathcal{I D} \mathcal{L}$

```
Algorithm 1 M-Instantiation
    procedure StandardInput::InstantiatedTerms::M-Instantiate
    for term \(\in\) terms do
        new-term \(\leftarrow\) (term +1 ).simplify ()
        if \(\neg\) inSet(new-term, terms) then
                terms.push-back(new-term)
            end if
            new-term \(\leftarrow\) (term - 1).simplify ()
            if \(\neg\) inSet(new-term, terms) then
                terms.push-back(new-term)
            end if
    end for
    end procedure
```


## Interpolation Algorithm

```
Algorithm 2 Main Loop
    : procedure AXDInterpolator::MAInLoop(StandardPair part-a, StandardPair part-b)
    if }\neg\mathrm{ (common-array-vars.areCommonPairsAvailable()) then
        SmtSolverSetup(solver, part-a)
        SmtSolverSetup(solver, part-b)
        if solver.check() = z3::unsat then
            is-unsat }\leftarrow\mathrm{ true
        end if
        return
        end if
        CircularPairlterator search-common-pairs(common-array-vars)
        while (num-attemps++ < remaining-fuel) do
            solver.push()
            SmtSolverSetup(solver, part-a)
            SmtSolverSetup(solver, part-b)
            if solver.check() = z3::unsat then
            is-unsat }\leftarrow\mathrm{ true
            return
            end if
            solver.pop()
            common-pair }\leftarrow*\mathrm{ *search-common-pair
            part-a-dim \leftarrow part-a.diff-map.size-of-entry(common-pair)
            part-b-dim }\leftarrow\mathrm{ part-b.diff-map.size-of-entry(common-pair)
            dim}\leftarrow\operatorname{min}(\mathrm{ part-a-dim, part-b-dim)
            new-index = fresh-index-constant()
            part-a.updateSaturation(common-pair, new-index, dim)
            part-b.updateSaturation(common-pair, new-index, dim)
            part-b.updateSaturation(com
        search-c
end procedure
```


## Algorithm 3 SmtSolverSetup

procedure AXDInterpolator::SmTSolverSetup(z3::solver solver, StandardPair side-part)
for assertion $\in$ side-part.part-2 do
solver.add(assertion)

## end for

side-part.instantiate(solver, $\forall x, i . i<0 \rightarrow r d(x, i)=\perp$ )
side-part.instantiate(solver, $\forall i . r d(\varepsilon, i)=\perp$ )
for $a=w r(b, i, e) \in$ side-part.write-vector do
side-part.instantiate(solver, $\forall h . h \neq i \rightarrow \operatorname{rd}(a, h)=r d(b, h))$
end for
for $\operatorname{diff}(a, b)=i \in$ side-part.diff-map do
side-part.instantiate(solver, $\forall h . h>i \rightarrow r d(a, h)=r d(b, h))$
end for
end procedure

procedure StandardPair::UPDATESATURATION(z3Pair entry, z3::expr new-index, unsigned min-dim)

$$
a \leftarrow \text { entry.first }
$$

$\mathrm{b} \leftarrow$ entry.second
map-element $\leftarrow$ diff-map.find(entry)
instantiated-terms.addVar(new-index)
if Heuristic then
end if
part-2.push-back(new-index $=$ (map-element.second) $[$ min-dim]
prev-index $\leftarrow$ (map-element.second)[old-min - 1
part-2.push-back(prev-index $\geq$ new-index)
part-2.push-back(new-index $\geq 0$ )
part-2. push-back(prev-index $=$ new-index $\rightarrow$ prev-index $=0$ )
end if
end procedure

## Architecture Overview



## Input Files and Extended Signature

Our extended language is parameterized by the array sorts in the input formula as well as by an index theory. The domain sort of every array is currently implemented using the Int sort.

```
(declare-sort A)
(declare-fun diff'A' ((Array Int A) (Array Int A)) Int)
(declare-fun length'A' ((Array Int A)) Int)
(declare-fun empty_array'A' () (Array Int A)
(declare-fun undefined'A' () A)
```

Figure: Our extended language parameterized with a sort $A$.

Demo

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## Benchmarks using SV-COMP and UAutomizer - Setup

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## Benchmarks using SV-COMP and UAutomizer - Setup

- We tested our implementation using C-programs from the ReachSafety-Arrays and MemSafety-Arrays tracks of the SV-COMP [3]
- We used the model checker UAutomizer [4] to extract their SMT Scripts from the previous C-programs
- We let the machine produce SMT Scripts for 15 minutes. We used these SMT Scripts files to compare the number of interpolants computed from unsatisfiable formulas. For the latter we assigned each process up to 360 seconds and 6 GB of memory


## Benchmarks using SV-COMP and UAutomizer - Memsafety-track Results

|  | AXD Interpolator |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Subtracks | IZ3 |  | MathSAT |  | SMTInterpol |  |
|  | Success | Timeout | Success | Timeout | Success | Timeout |
| array-examples | 584 | 1 | 584 | 1 | 584 | 1 |
| array-memsafety | 118 | 0 | 118 | 0 | 118 | 0 |
| termination-crafted | 52 | 3 | 52 | 3 | 52 | 3 |

Table: Memsafety-track results - Our implementation

| Subtracks | iZ3 |  | MathSAT |  | SMTInterpol |  |
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| array-examples | 585 | 0 | 585 | 0 | 585 | 0 |
| array-memsafety | 118 | 0 | 118 | 0 | 118 | 0 |
| termination-crafted | 55 | 0 | 55 | 0 | 55 | 0 |

Table: Memsafety-track results - Other solvers

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| Subtracks | iZ3 |  |  | MathSAT |  |  |  | SMTInterpol |
|  | Success | Timeout | Success | Timeout | Success | Timeout |  |  |
| array-cav19 | 31 | 0 | 31 | 0 | 31 | 0 |  |  |
| array-examples | 50 | 0 | 50 | 0 | 50 | 0 |  |  |
| array-fpi | 774 | 21 | 774 | 21 | 774 | 21 |  |  |
| array-industry-pattern | 8 | 0 | 8 | 0 | 8 | 0 |  |  |
| array-lopstr16 | 54 | 0 | 54 | 0 | 54 | 0 |  |  |
| array-patterns | 11 | 0 | 11 | 0 | 11 | 0 |  |  |
| array-tiling | 6 | 0 | 6 | 0 | 6 | 0 |  |  |
| reducercommutativity | 53 | 0 | 53 | 0 | 53 | 0 |  |  |

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|  | Success | Timeout | Success | Timeout | Success | Timeout |
| array-cav19 | 31 | 0 | 31 | 0 | 31 | 0 |
| array-examples | 50 | 0 | 50 | 0 | 50 | 0 |
| array-fpi | 795 | 0 | 795 | 0 | 795 | 0 |
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Table: Reachsafety-track results - Other Solvers

## Future Work

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- The current design does not perform incremental satisfiability checks. Incremental checks are possible to implement due to the incremental nature of the proposed interpolation algorithm by including a hash consed data structure on the terms/predicates produced in the main loop of the algorithm and because the data structure z3: :solver can keep track of previously proven assertions.


## Conclusions

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- We were able to show the feasibility of AXDInterpolator by validating it on two benchmarks taken from the SV-COMP.
- We also compared our implementation with state-of-the-art solvers: apart from very few timeout outcomes, our tool managed to handle all the examples the other solvers did.
- We also found interesting examples that are not handled by other state-of-the-art solvers, which makes the option of our language extension and tool an appealing consideration.


## Thanks for your attention!

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