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REPRESENTING CONCEPTS AND INFERENCING IN APPLIED MATHS

A proposal for a paper at CAL 77 by

George F. Luger
Research Fellow
Department of Artificial Intelligenc
University of Edinburgh
Edinburgh Scotland EH8 9NW

Abstract

This paper is divided into two parts. In the first half the MECHO* project will be described; in the final half several implications of this research for modelling and teaching concepts in applied maths and engineering will be outlined.

The MECHO project (MECHanics Oracle) consists of designing and implementing a computer program to solve mechanics problems stated in English. This work is motivated by a desire to understand how the representation of a problem changes in the process of its solution. In particular how a natural language statement is changed to a set of equations in the light of an abstract model of the real world situation; and finally how a solution is derived from this set of equations.

As the project has matured the design for problem solving seems to demand an interactive approach. For example, rather than a natural language parse that attempts to produce all information contained in the problem statement, a particular model of the situation might aid in extracting the relevant information necessary to extract a particular set of equations. Also, "real world" knowledge in a problem can eliminate extraneous roots (such as negative periods of time or imaginary roots) from the solution of a set of simultaneous equations.

In the course of the project various "sources" of problem solving data have been consulted. Besides the works of Polya and earlier attempts in this same domain (Charniak, Bobrow, and Novak) protocols of problem solving subjects and the introspections of the researchers were employed. Finally, related projects such as the Marples & Simpson study of problem solving in the applied maths in the Engineering Department at Cambridge were a valuable source of materials.

In the second section of the paper the traditional approaches to teaching mechanics are compared with a more model theoretic view. In many contemporary educational situations the applied maths or engineering

studies are presented as sets of loosely related problems. In a typical unit a short introduction (for example, on "Newton's Laws of Motion" - including a paragraph on "Force", one on "Momentum" and one on "Friction") may be followed by five or six worked out examples and then 25-30 often heterogeneous problems. A tutor will often supply "cribs" for each of the set of problems assigned in a unit and the student will be expected to somehow produce his own solution to new "related" problems in an examination.

The MECHO approach would suggest a much more explicit presentation of the concepts and abstract models that lie beneath and motivate a set of equations as the representation of a real world situation. This presentation has a natural division into two sections:

1. The study of the concepts and models themselves, for example of pulley systems constructed of extensible or inextensible strings, rough or smooth pulleys, wedges, tables, and so on, and
2. The production from these models of sets of simultaneous equations sufficient for the solution of the problems.

Each of the points above will be more fully exemplified by considering three problems from the world of pulley systems. These problems will demonstrate the depth and sophistication the pulley system model must contain if it is to solve the problems. Furthermore, the problems will characterize the "abstractions" the problem solver must use to call up and implement a model of a situation. For example, when a person in a problem is to be considered as a point mass or when as the Source of a force.

In conclusion the MECHO study of the changes in representation necessary in the computer solution of problems in applied maths and engineering will be used to suggest an emphasis for learning in these areas, whether it be in the traditional classroom setting or the newer environment of computer assisted learning. Finally, it it hoped that both the concepts and models developed and the equation extraction and solution procedures implemented in the MECHO project will offer the builders of interactive computer learning systems a source of ideas and motivations for their research.

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REPRESENTING SEMANTIC INFORMATION IN PULLEY PROBLEMS

George F. Luger & Alan Bundy
Department of Artificial Intelligence
University of Edinburgh
Scotland

Several recent research reports have focussed on the representation and solution of problems in semantically rich domains (1),(2),(3),(4). These include analysis of algebra word problems as well as problems in mechanics and thermodynamics. The term "semantically rich" characterizes these domains well in that successful problem solvers possess large amounts of problem specific information and task related knowledge. Compare this with the knowledge and information needed to solve tasks such as the Tower of Hanoi or cryptarithmetic.

The MECHO group (2) has employed the familiar protocol analysis technique in an attempt to determine what information the person attempting to solve pulley problems has available, and further, how this information is organized and stored. Although the MECHO project has much broader goals than characterizing the world of pulley problems, the representation and solution of pulley problems will be the focus of this paper.

Like (1) & (3) we employ a problem-type schema to represent the problem solver's knowledge. We are also attempting to describe the content of the problem-type schema and characterize this content by a computer program (4).

The term schema is used, after Bartlett and Piaget, to refer to a structuring of information, a loose condederation of relationships that represent a capacity to perform a function or task. Each schema is made up of sets of facts and relations about an ideal object configuration, a set of inferences about these facts and relations, and a set of default values.

Consider the pulley problem: A man of 12 stone and a weight of 10 stone are connected by a light rope passing over a pulley. Find the acceleration of the man. If the man pulls himself up the rope so that his acceleration is one half its former value, what is the acceleration of the weight?

The facts or declaration of the problem above include a fixed pulley, a light rope passing over the pulley, and two objects one at each end of the rope.

The inferences about the pulley system will include the fact that both halves of the string will have the same tension if the pulley is smooth. Also, the magnitude of the acceleration of each object will be the same - the directions of course, are different - if the length of the rope between the objects remains constant. This acceleration inference is necessary if the solver is able to answer both questions in the problem. In the first instance, with the length of rope between the objects constant, the acceleration of the objects will be equal - though in opposite directions. In the second instance, the length of rope between objects changes so the magnitude of the acceleration will be different for each object.

The default values of the problem schema contain information such as, if nothing is said to the contrary, the pulley will be light and frictionless. Also, it is assumed the pulley is fixed if nothing is said to the contrary.

The following pulley system schema (in PROLOG (5)) will partially represent this situation:

```
Schema (pullsys (_sys,_pulley ,__string,_directionl,
      _direction2,__time) ,
  {cue(stringsys(_string,_leftend,_jnidpt,_rightend,
    _time))3,
  [problemtype (__pulley) ,particle (jpulley) ,
    incline(_leftend,_directionl,_midpt),
    incline(__rightend,_direction2,_midpt),
    tension (_leftend,_T,__time) :-  friction (_pulley,
      zero), tension( string,_T, __time)),
    tension(_rightend,_T,_time):-  friction
      (__pulley,zero) ,tension (_string, _T,__time)) ] ,
  [friction(_pulley,zero), mass(_pulley,zero, time)
    fixed__contact (___pulley,_earth,_time)   :-
      refpoint(_pt), diff(jpt,earth),
      nlc (thnot (fixed__contact (_pulley,_j>t,_time) ) )]).
```

Each schema, as above, consists of four parts:
(l)the key, e.g., pullsys(....) used for calling in the appropriate schema.
(2)the <u>declaration</u>, e.g., cue(stringsys(...) list of subgoals for linking this schema to information in the database or declaring new information.
(3)the <u>assertions</u>, e.g., tension(...) :- (....) for adding facts or inferences to the database.
(4)the <u>defaults</u>, e.g., friction(...) a list of facts or inferences asserted as default values.

The problem schema represents the condition driven aspect of the MECHO program. It asserts the facts,inferences and defaults that make up the semantic content of the problem domain. The MECHO system is also goal driven. A simple means-ends analysis is incorporated in an algorithm that takes a sought unknown (the acceleration of the man) and by creating intermediate unknowns (the tension in the string) ~~generates a~~ linearly independent set of equations that will solve the problem. Thus MECHO is both a condition and goal driven system.

The problem-type schemata outlined in this paper are developed further in (4). There human protocols are presented and used to justify the pulley problem schemata. The MECHO system is also discussed as a model of human performance in the solution of pulley problems.

<u>References</u>

(l)Baskar,R. & Simon,H. Problem solving in semantically rich domains: an example from engineering thermodynamics, COGNITIVE SCIENCE,1,2, 1977.
(2)Bundy,A.,Luger,G.,Stone,M. & Welham,R. MECHO: year one,Research Report 22,Dept.of Artificial Intelligence,Edinburgh, 1976.
(3)Hinsley,D, Hayes,J, & Simon,H, From words to equations: meaning and representation in algebra word problems,CIP 33,Dept.of Psychology,C-M U, 1976.
(4)Luger,G. Protocols and the representation of information in pulley problems,Research Report, Dept. of Artificial Intelligence, Edinburgh,1977.
(5)Warren,D. Epilog: users guide to DEC10 PROLOG, Dept. of Artificial Intelligence, Edinburgh,1975.

Luger, G.F. and Bundy, A. (1977). Representing Semantic Information in Pulley Problems. In *Proceedings of Fifth International Joint Conference in Artificial Intelligence.* Los Altos: Morgan Kaufmann.