1. Let $p$ and $q$ be the propositions “You are an engineer” and “You love plaid” respectively. Express each of the following compound propositions as an English sentence.

(a) $\neg p$
(b) $\neg p \land q$
(c) $\neg q \Rightarrow \neg p$
(d) $\neg q \lor (\neg p \land q)$

2. Let $p$, $q$ and $r$ be the following propositions: “You are a mathematician”; “You are an engineer”; and “You are a sharp dresser” respectively. Write these propositions using $p$, $q$ and $r$ and logical connectives.

(a) You are a mathematician but you are not an engineer
(b) You are a mathematician, you are an engineer and you are a sharp dresser
(c) If you are a sharp dresser, you must be a mathematician
(d) Being a mathematician and an engineer is sufficient to be a sharp dresser
(e) You are a sharp dresser if and only if you are a mathematician or you are an engineer

3. Exercise 1-17-24: State the converse, contrapositive and inverse of each of these conditional statements

(a) If it snows tonight, then I will stay home
(b) I go to the beach whenever it is a sunny day
(c) When I stay up late, it is necessary that I sleep until noon
4. You are on an island where all people are either truth tellers, who always tell the truth, or liars, who never tell the truth. A person on this island is accused of a crime, and hires an attorney. The defendant is publicly known to be a truth teller. The following exchange takes place in court:

Attorney: “If the defendant committed the crime, he had an accomplice.”

Defendant: “That is not true!”

Did the attorney help his client? Justify your answer.

5. What if the Attorney was publicly known to be a liar and she says “If the defendant committed the crime, he did not have an accomplice.”

Does the attorney help or hurt his client, or neither? Justify your answer. Hint: Let \( p \) be the proposition “The defendant committed the crime” and \( q \) be the proposition “The defendant had an accomplice”. Use De Morgan’s rule!

6. Show that \( p \Rightarrow q \) and \( \neg q \Rightarrow \neg p \) are logically equivalent using either truth tables or rules of logical equivalence (p. 24)

7. Show that \( ((p \lor q) \land (\neg p \lor r)) \Rightarrow (q \lor r) \) is a tautology.

8. How many of the following disjunctions can be made simultaneously true by an assignment of truth values to \( p, q \) and \( r \): \( p \lor q \), \( \neg p \lor r \), \( \neg p \lor \neg q \), \( \neg r \lor p \).

9. Exercise 1.3.6

10. Exercise 1.3.38

11. Exercise 1.3.42

12. You are lounging on the beach on the island of liars and truth tellers with a large group of natives and you hear the following exchange:

Alice: “We are all liars and Bob is a truth teller”;

Bob: “We are all liars or Carol is a liar”

What can you say about Alice, Bob and Carol? Justify your answer. Hint: Let \( L(x) \) be the proposition that \( x \) is a liar; use quantifiers and De Morgan’s laws for quantifiers in your answer. Consider the two cases where Alice is a truth teller or Alice is a liar.

13. Challenge: You are investigating a murder on the island of liars and truth tellers. You have assembled a group and you want to know
if the murderer is in that group. You know that the murderer is a liar. However, you don’t know which members of the group, if any are truth tellers. Moreover, you are only allowed to ask yes/no questions to the leader of the group and you want to minimize the number of questions you ask. Hint: Your questions may use propositional logic and quantifiers. Let $M(x)$ be the proposition, “person $x$ is a murderer in the group”. Let $L(x)$ be the proposition “person $x$ is a liar”.

- Show how to determine if the murderer is in the group by asking two yes/no questions to the group leader.
- Show how to determine if the murderer is in the group by asking a single yes or no question to the leader. Hint: For a person $x$, let $\text{Say} – \text{Murderer}(x)$ be a proposition that is true if $x$ would say that there is a murderer in the group.