CS 261, HW2

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Due: Feb. 10th

1. Show that $\forall x, P(x) \vee \forall x Q(x)$ and $\forall x, (P(x) \vee Q(x)$ are not logically equivalent

2. Let P(x), Q(x) and R(x) be the statements"x likes wood-elves", "x favors plaids", "x has floppy ears "

   - Everybody who likes wood-elves favors plaids
   - Somebody who likes wood-elves does not favor plaids
   - Nobody with floppy ears likes wood-elves

3. Give the negation of each of your statements in the previous question and rewrite these statements so that 1) there are no $\Rightarrow$ symbols and 2) the negations appear only within the predicates.

4. Let $Q(x, y)$ be the statement $x = y^2$. Give the truth value of the following statements over the integers

   - $Q(0, 0)$
   - $\forall x, \exists y, Q(x, y)$
   - $\forall x, \exists y, Q(y, x)$
   - $\exists x, \forall y, Q(x, y)$
   - $\exists y, \forall x, Q(x, y)$
   - $\exists x, \exists y, Q(x, y)$

5. Let $P(x), Q(x), R(x), S(x, y)$ be the predicates, "x is a true dungeon master", "x has Max-Charisma", "x is a wood-elf", "x is a friend to y". Translate the following statements into predicate logic.

- A true dungeon master is a friend to all wood-elves
- Only true dungeon masters have Max-Charisma
- Bob is not a friend to some wood-elf

6. Using the statements from the above problem, prove that Bob does not have Max-Charisma. Justify every line of your proof with a rule of logic as in the proofs in the text.
7. Prove that if x is an odd integer, then $(x+1)^2$ is an even integer
8. Prove that if x^2+1 is odd, then x is even (hint: contrapositive)
9. Exercise 1.5.16
10. Prove that $2^{1/3}$ is irrational
11. Exercise 1.7.40
12. Prove or disprove that you can use dominoes to tile a 5 by 5 checkerboard
13. Exercise 1.7.42