CS 261 HW7

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Due May 3rd

1. Prove that for any constants a and b, where $a > b$, that $a^n + b^n$ is $O(a^n)$.
2. Solve the following recurrence relation with annihilators and a change of variables: $f(n) = f(n/2) + f(n/4)$.
3. Write and solve a recurrence relation for the number of bit strings that contain the string "00". Hint: One way to get such a sting is to append 00 to the end of any bit string of length $n-2$. What are the other ways to get such a string (think recursively!)
4. Exercise 7.1.48 parts a and b only. (Read discussion of tower of hanoi on page 452 and 453 first) Hint: Think recursively about how you could use a subroutine that moves all of the disks but the largest disk from peg 1 to peg 3.
5. Exercise 7.1.50 (Note: You should also try to solve 7.1.49 and 7.1.51 yourself, but since the answers are in the back of the book, you are not required to turn them in)
6. Exercise 7.1.52
7. Many important numbers, like credit card numbers and social security numbers, have validation checks to make it harder for a criminal to randomly generate them. Imagine that after you get your CS degree, you begin work at a criminal syndicate that is trying to create random numbers that will be accepted as valid frequent flyer numbers. Imagine that for a certain Estonian airline, a valid frequent flyer number obeys the following rules: 1) it is a n digit binary number; 2) it does not contain the string "00"

   In this problem, you will calculate the probability that a randomly generated number is a valid frequent flyer number for Air Estonia.

(a) Write a recurrence relation for the number of valid frequent flyer numbers of length n

(b) Solve the above recurrence relation

(c) What is the number of unconstrained n digit binary numbers

(d) Using the solutions to the above two problems, give the probability that a random n digit number is a valid frequent flyer number? What is a good approximation to this probability for large n?

8. Exercise 6.1.35

9. Exercise 6.1.40 (Monty Hall puzzle - read p. 398 first)

10. Use indicator variables to solve the following problem. Each of n dogs in a doggie spa has its name on a specific dog house. After a long walk one day, the dogs are brought back to the spa and each dog runs to a random dog house (without regards to how many dogs may already be in that house). What is the expected number of dogs that actually wind up in their own dog house?

11. How many people must be in a room before you would expect 3 of them to share the same birthday?