CS261, Spring '19, Midterm Practice Problems

1 Final Review Problems

1. Exercise 4.4.39
2. Exercise 4.4.40
3. Exercise 4.4.65
4. Exercise 4.6.25
5. Exercise 4.6.27
6. Exercise 4.6.31
7. Exercise 5.1.68
8. Prove that given an unlimited supply of 6-cent coins, 10-cent coins, and 15-cent coins, one can make any amount of change larger than 29 cents. Hint: Induction.
9. Prove that $n \geq 1$ lines in the plane passing through the same point divide the plane into $2n$ regions. Hint: Induction.
10. Exercise 5.1.68 "Finding a Celebrity" problem.
11. Prove that $5|(6^n + 4)$ for all $n \geq 0$. Let your IH be that for all $0 \leq k < n$, $5|(6^k + 4)$.
12. Find the number of subsets of $S = \{1, 2, \ldots 10\}$ that contain 5 elements, two of which are 3 and 4.
13. Find the number of subsets of $S = \{1, 2, \ldots 10\}$ that contain 5 elements, each including 3 or 4 but not both.

    A class consists of 20 sophomores and 15 freshmen. The class needs to form a committee of size five.

(a) How many committees are possible?

(b) How many committees are possible if the committee must have three sophomores and two freshmen?

14. Suppose $|A| = 4$ and $|B| = 10$. Find the number of functions $f : A \to B$.

15. Suppose $|A| = 4$ and $|B| = 10$. Find the number of 1-1 functions $f : A \to B$.

16. How many bit strings of length 10 have equal numbers of 0's and 1's?

17. A computer randomly prints three-digit codes, with no repeated digits in any code (for example, 387, 072, 760). How many unique codes are possible?

18. What is the minimum number of codes that must be printed in order to guarantee that at least six of the codes are identical?

19. A computer network consists of six computers. Each computer is directly connected to zero or more of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of computers.

20. Urn 1 contains 2 blue tokens and 8 red tokens; urn 2 contains 12 blue tokens and 3 red tokens. You pick an urn at random and draw out a token at random from that urn. Given that the token is blue, what is the probability that the token came from urn 1?

21. 25 independent fair coins are tossed in a row. What is the expected number of consecutive HH pairs? Hint: Linearity of Expectation!

22. For problems on the Master Method and Annihilators, see HW 9 problems 4 and 5, and my 2010 final exam, problem 3 at: https://www.cs.unm.edu/ saia/classes/261-s19/final.pdf

2 Midterm Review Problems

1. Prove the proposition "if it is not hot, then it is hot" is equivalent to "it is hot".

2. Prove that $p \to q$ and its inverse are not logically equivalent.

3. Prove that $p \to q$ and its converse are not logically equivalent.

4. Write the contrapositive, converse and inverse of the following: "You will eat it, if it is bacon."

5. Is $((p \to q) \wedge \neg p) \to \neg q$ a tautology? Prove your answer!

6. Let x and y be real numbers and let $L(x, y) : x < y$; $G(x) : x > 0$; $P(x) : x$ is a prime number. Write the following statements in English without using any variables.

   (a) $L(3, 7)$

   (b) $\forall x \exists y L(x, y)$

   (c) $\forall x \exists y [G(x) \to (P(y) \wedge L(x, y))]$

7. Now translate the following sentences into predicate logic using the functions from the last problem.

   (a) Every prime has some prime that is larger than it.

   (b) There is a real number between any two primes.

   (c) It is not the case that there is a prime between any two real numbers.

8. Chapter 1, Review Question 10 (Direct proof, Proof by Contradiction, Proof by Contraposition); Supplementary Exercises 37, 38, 40.

9. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

10. Chapter 2: 2.3.21, 2.3.22, 2.4.17, 2.4.23

11. Prove that the set $(0, 1)$ has the same cardinality as $\mathbf{R}$. Recall that (x, y) is the set of all real numbers greater than x and less than y, and $\mathbf{R}$ is the set of all real numbers. Hint: Show a one-to-one correspondence between $(0, 1)$ and $\mathbf{R}$.

12. Give a recurrence relation that generates the following sequences

    (a) $5, 8, 11, 14, \ldots$

    (b) $2, 0, 2, 0, 2, 0, \ldots$

    (c) $1, 11, 111, 1111, \ldots$

    (d) $1, 2, 4, 8, \ldots$

13. Solve the recurrence $a_n = 3a_{n-1} + 1$, and $a_0 = 1$.

14. Find the sum $1/4 + 1/8 + 1/16 + \ldots$

15. Give the best big-O complexity of the following algorithms. Choose from the list $1, \log n, n, n \log n, n^2, 2^n, n!$.

    (a) A binary search of n elements.

    (b) A linear search to find the smallest number in a list of n numbers.

    (c) An algorithm that lists all ways to put the numbers $1, 2, 3, ..., n$ in a row.

    (d) An algorithm that prints all bit strings of length n.

    (e) An algorithm that finds the average of n numbers.

16. Show that n^3 is not $O(n^2)$. Hint: Use the definition of big-O.

17. Exercises 3.3.20 and 3.4.11

18. Prove or disprove: For all integers a, b, c, d, if $a|b$ and $c|d$, then $(ac)|(b+d)$.

19. Prove or disprove: If $a \equiv b \pmod{m}$ then $2a \equiv 2b \pmod{2m}$.

20. Given that gcd(662, 414) = 2, write 2 as a linear combination of 662 and 414.

21. Solve the linear congruence $2x \equiv 5 \pmod{9}$.

22. Find an inverse of 5 modulo 17 using Extended Euclid.

23. Solve the linear congruence $5x \equiv 2 \pmod{17}$. Hint: Use the answer from the last problem!