CS 261, HW2

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Due: Tuesday, Feb. 12th

1. Show that $(\forall x P(x)) \vee (\forall x Q(x))$ and $\forall x (P(x) \vee Q(x))$ are not logically equivalent

2. Let P(x), Q(x) and R(x) be the statements "x likes wood-elves", "x favors plaids", "x has floppy ears". Translate the following sentences into predicate logic.

   (a) Everybody who likes wood-elves favors plaids

   (b) Somebody who likes wood-elves does not favor plaids

   (c) Nobody with floppy ears likes wood-elves

3. Give the negation of each of your statements in the previous question and rewrite these statements so that 1) there are no $\Rightarrow$ symbols and 2) the negations appear only within the predicates.

4. Let $Q(x, y)$ be the statement $x = y^2$. Give the truth value of the following statements over the integers

   - $Q(0, 0)$
   - $\forall x, \exists y, Q(x, y)$
   - $\forall x, \exists y, Q(y, x)$
   - $\exists x, \forall y, Q(x, y)$
   - $\exists y, \forall x, Q(x, y)$
   - $\exists x, \exists y, Q(x, y)$

5. Let $P(x), Q(x), R(x), S(x, y)$ be the predicates, "x is a true dungeon master", "x has Max-Charisma", "x is a wood-elf", "x is a friend to y". Translate the following statements into predicate logic.

- A dungeon master is a friend to all wood-elves
- Only dungeon masters have Max-Charisma
- Bob is not a friend to some wood-elf

6. Using the statements from the above problem, prove that Bob does not have Max-Charisma. Justify every line of your proof with a rule of logic as in the proofs in the text.

7. Prove that if x is an odd integer, then $(x + 1)^2$ is an even integer

8. Prove that if $x^2 + 1$ is odd, then x is even (hint: contrapositive)

9. Exercise 1.5.16 (starting "A Discrete Mathematics class contains ...")

10. Prove that $2^{1/3}$ is irrational

11. Prove that if x^3 is irrational, then x is irrational.

12. Exercise 1.7.40 ("Use Exercise 39 to show that if...". Note that the solution to Exercise 1.7.39 is in the back of the book.)

13. Prove or disprove that you can use dominoes to tile a 5 by 5 checker-board