

## CS 261, HW3

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*Due: Feb. 21*

This homework covers material from Chapter 2.2 up to and including Chapter 3.1 in the textbook.

1. Prove that if  $A$  and  $B$  are sets then  $(A \cap B) \cup (A \cap \bar{B}) = A$ . Hint: Show two things: (1) if  $x$  is in  $(A \cap B) \cup (A \cap \bar{B})$ , then  $x$  is in  $A$ ; and (2) if  $x$  is in  $A$  then  $x$  is in  $(A \cap B) \cup (A \cap \bar{B})$ .
2. Give an example of a function from the integers, to the integers that is
  - (a) One-to-one but not onto
  - (b) Onto but not one-to-one
  - (c) Both onto and one-to-one, but NOT the identity function
  - (d) Neither one-to-one or onto
3. Determine if each of the following functions is a bijection from the Reals to the Reals.
  - (a)  $f(x) = -2x + 4$
  - (b)  $f(x) = x^2 + 9$
  - (c)  $f(x) = x/(x + 1)$
  - (d)  $f(x) = x^7 + 2$
4. What are the values of the following sums?
  - (a)  $\sum_{i=1}^{10} (i + 1)$
  - (b)  $\sum_{i=1}^{10} (-2)^i$
  - (c)  $\sum_{i=1}^{10} 4$
5. What are the values of the following products

- (a)  $\prod_{i=0}^5 i$
- (b)  $\prod_{i=4}^7 i$
- (c)  $\prod_{i=4}^7 2$
- (d)  $\prod_{i=1}^{20} (-1)^i$

- 6. Exercise 2.4.10: “Find the first six terms of the sequence ...”
- 7. Exercise 2.4.16: “Find the solution to each of these recurrence relations ...”
- 8. Consider the set of all tuples of the form  $(x, y)$  where  $x$  and  $y$  are positive integers. Prove that this set is countable.
- 9. Consider the set of all strings of DNA, where a DNA string may be infinite in length. Prove that this set is not countable. A DNA string is a sequence made up of the four letters A, C, T and G, of any length, including infinite. *Hint: Review the proof that the real numbers are not countable*
- 10. Exercise 2.5.4: “Determine whether each of these sets is countable or uncountable...”