CS 261 HW6

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Due Thursday, April 11th

This homework covers material from Chapter 5.

1. Exercise 5.2.4 (4 cent and 7 cent stamps)

2. Exercise 5.2.10 (breaking a chocolate bar)

3. The game of Chomp is described in Chapter 1.8, Example 12. Prove, using strong induction, that the first player has a winning strategy in the game of Chomp if the board is 2 by n, i.e. each row has 2 cookies. Hint: Use strong induction and let the first move be to chomp the bottom right cookie. Then show that if a player takes a turn where they have a 2 by n rectangle with the bottom cookie removed, that player can be forced to always eventually lose.

4. Consider the matchstick game in Chapter 5.2, Example 3. In this game, there are 2 piles, each with n matches. Each player can take any number of matches from a pile and the player who removes the last match wins. In the book, they show that the second player can always win this simple game.
   In this problem, you will consider the case where there are x piles, each containing n matches, where x is an even number. Prove that the second player can always win in this game. Hint 1: You will find it easier to prove something stronger: if the number of matches in the k and $k+1$ piles are the same for all odd k between 1 and $x-1$, that the second player can win. Hint2: Think of a strategy that allows you to use strong induction on the number of piles.

5. Consider the recurrence relation for the Fibonacci numbers that we discussed in class: $f(n) = f(n-1) + f(n-2)$, $f(1) = f(2) = 1$. Prove using induction that for all $n \geq 2$, $f(n) \geq (3/2)^{n-2}$.

6. Let $f(n)$ be the number of ways to tile a 2 by n rectangle with 2 by 1 dominoes. Note that $f(0) = 1$, $f(1) = 1$ and $f(2) = 2$. Write a recurrence relation for $f(n)$ for general n. What type of numbers discussed in class are the solutions to your recurrence relation?

7. Give a recursive algorithm to reverse a string s. Note: The reversal of "a" is "a". The reversal of "ab" is "ba". The reversal of "bba" is "abb".

8. Prove that your reversal algorithm from problem 8 is correct using induction.

9. Give a recursive algorithm for finding $2^n \bmod m$.

10. Supplementary Exercise 34: "Use mathematical induction to show that if you draw lines in the plane you only need two colors to color the regions formed so that no two regions that have an edge in common have a common color."