# CS 261 HW7

### Prof. Jared Saia, University of New Mexico

### Due April 18th

This homework covers material from Chapter 6.1 up to and including Chapter 6.3 in the textbook.

1. Imagine that Mergesort is changed so that it partitions the input list into 3 sub-lists instead of just 2, that it recursively calls Mergesort on each of these three lists and that it then calls a Merge subroutine on the 3 sorted lists to merge them into a single sorted list. Assume that this Merge subroutine works in $n$ time where $n$ is the total number of elements in all 3 lists. Write and solve (using induction) a recurrence relation for the run time of this new version of Mergesort.

2. A person deposits \$1 in a savings account that yields 8% interest annually, and then never touches the account again. Thus, at the end of every year, the bank adds to the account an amount equal to 8% of the account value during that year. Set up a recurrence relation for the amount in the account at the end of $n$ years. Now find an explicit formula for the amount in the account after $n$ years.

3. Exercise 6.1.32 ("How many letters of 8 uppercase...")

4. How many people must be in a room before you can be sure that 3 of them have the same birthday (assume 366 possible days in the year)?

5. Exercise 6.2.12 ("How many ordered pairs of integers...")

6. The numbers 0 through 99 are placed in a hat and 51 of them are drawn. Prove that there must be some pair of drawn numbers $x$ and $y$ such that $x \equiv y + 7 \pmod{100}$. Hint 1: For $1 \leq i \leq 51$, let $x_i$ be the $i$-th number drawn. For $1 \leq i \leq 51$, consider all pairs $(x_i, (x_i + 7) \bmod 100)$. If the above statement doesn't hold, can any of these pairs intersect?

7. On the CS261 final exam, there are 26 True/False questions. Exactly 14 of these questions are True. If these questions can be put in any order, then how many different answer keys are possible?

8. There is also a matching section on the exam, where each of 10 questions need to be matched with one of 10 answers, and each answer is used for exactly one question. How many different answer keys are there for this section? What if question Q1 must match with answer A1, and question Q10 must match with answer A10. How many different answer keys are there now?

9. Exercise 6.3.40 "Find the number of circular 3-permutations of 5 people"

10. Exercise 6.3.42 "Find a formula for the number of ways to seat r of n people around a circular table..."