CS261, Spring '19, Midterm Practice Problems

1. Prove the proposition "if it is not hot, then it is hot" is equivalent to "it is hot".
2. Prove that $p \to q$ and its inverse are not logically equivalent.
3. Prove that $p \to q$ and its converse are not logically equivalent.
4. Write the contrapositive, converse and inverse of the following: "You will eat it, if it is bacon."
5. Is $((p \to q) \land \neg p) \to \neg q$ a tautology? Prove your answer!
6. Let x and y be real numbers and let $L(x, y) : x < y$; $G(x) : x > 0$; $P(x) : x$ is a prime number. Write the following statements in English without using any variables.
   (a) $L(3, 7)$
   (b) $\forall x \exists y L(x, y)$
   (c) $\forall x \exists y [G(x) \to (P(y) \land L(x, y))]$
7. Now translate the following sentences into predicate logic using the functions from the last problem.
   (a) Every prime has some prime that is larger than it.
   (b) There is a real number between any two primes.
   (c) It is not the case that there is a prime between any two real numbers.
8. Chapter 1, Review Question 10 (Direct proof, Proof by Contradiction, Proof by Contraposition); Supplementary Exercises 37, 38, 40.
9. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.
10. Chapter 2: 2.3.21, 2.3.22, 2.4.17, 2.4.23

11. Prove that the set $(0, 1)$ has the same cardinality as $\mathbf{R}$. Recall that (x, y) is the set of all real numbers greater than x and less than y, and $\mathbf{R}$ is the set of all real numbers. Hint: Show a one-to-one correspondence between $(0, 1)$ and $\mathbf{R}$.

12. Give a recurrence relation that generates the following sequences

    (a) $5, 8, 11, 14, \ldots$
    (b) $2, 0, 2, 0, 2, 0, \ldots$
    (c) $1, 11, 111, 1111, \ldots$
    (d) $1, 2, 4, 8, \ldots$

13. Solve the recurrence $a_n = 3a_{n-1} + 1$, and $a_0 = 1$.

14. Find the sum $1/4 + 1/8 + 1/16 + \ldots$

15. Give the best big-O complexity of the following algorithms. Choose from the list $1, \log n, n, n \log n, n^2, 2^n, n!$.

    (a) A binary search of n elements.
    (b) A linear search to find the smallest number in a list of n numbers.
    (c) An algorithm that lists all ways to put the numbers $1, 2, 3, ..., n$ in a row.
    (d) An algorithm that prints all bit strings of length n.
    (e) An algorithm that finds the average of n numbers.

16. Show that n^3 is not $O(n^2)$. Hint: Use the definition of big-O.

17. Exercises 3.3.20 and 3.4.11

18. Prove or disprove: For all integers a, b, c, d, if $a|b$ and $c|d$, then $(ac)|(b+d)$.

19. Prove or disprove: If $a \equiv b \pmod{m}$ then $2a \equiv 2b \pmod{2m}$.

20. Given that gcd(662, 414) = 2, write 2 as a linear combination of 662 and 414.

21. Solve the linear congruence $2x \equiv 5 \pmod{9}$.

22. Find an inverse of 5 modulo 17 using Extended Euclid.

23. Solve the linear congruence $5x \equiv 2 \pmod{17}$. Hint: Use the answer from the last problem!