Cache Performance of Indexing Data Structures

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Abstract

The speed of computer processors is growing rapidly in comparison to the speed of DRAM chips. The cost of a cache miss, measured in processor clock cycles, is increasing exponentially, and this is quickly becoming a bottleneck for indexing in main memory. We study several indexing data structures on a simulated architecture and show that the relative performance of cache-conscious indexing structures is increasing with memory latency. In addition, we show that top-down algorithms for maintaining these structures reduce the total instruction count, leading to a modest improvement in execution time over the corresponding bottom-up algorithms.

1. Introduction

Note: All figures and tables appear at the end of this document due to a formatting problem.

The speed of computer processors is growing rapidly in comparison to the speed of DRAM chips. The processor clock cycle has been decreasing at a rate of roughly 70% every year, while the cycle time of common DRAM chips is decreasing by about 33% every 10 years [6]. These trends indicate that the cost of a cache miss, measured in processor clock cycles, is increasing at an exponential rate.

Data structures used for indexing in main memory databases that are not optimized for cache performance will suffer due to the increasing cost of main memory access. This problem has already been studied for indexing in secondary storage where the cost of magnetic storage has been problematic for a long time [3]. The effects of caches on indexing performance are comparatively new and have the following distinguishing characteristics: cache blocks are much smaller than disk pages, cache replacement policies are fixed and have limited associativity, and cache access times are much shorter than secondary storage, relative to the processor, so it is insufficient to simply equate a tree's depth with its access time as is done for secondary storage.

We study the main memory performance of a variety of existing techniques for maintaining balanced trees, including B^+ -trees [3], AVL trees, treaps [11], and two variants of the top-down deterministic skip list [9]. Experimental data is measured using both current hardware and the SimpleScalar tool set [1] for detailed simulation of a modern processor. Our results indicate that cache-conscious indexing data structures currently outperform their counterparts by approximately 48%. A factor of five (25) increase in memory latency increases the relative performance difference to 77% (84%).

Another aspect of balanced trees that is important to consider is their implementation difficulty. Munro et al. propose that deterministic skip lists using topdown operations are simpler to understand and implement than many of the alternatives [9]. Topdown operations perform rebalancing in a single, downward pass through the tree, while *bottom-up* operations search on the way down and rebalance on the way back up, requiring the use of a stack or parent pointers [5]. Top-down operations also lead to an average reduction of 31% in total instruction count for our experiments. Because of this reduction a cache-conscious version of the basic skip list currently outperforms the corresponding B⁺-tree by 12%, though this advantage will decrease with time as memory access time begins to dominate overall performance. Still, if the performance of top-down and bottom-up algorithms are comparable then the simplest implementation should be chosen.

2. Skip Lists

Skip lists were originally introduced by Pugh as a randomized alternative to search trees [10]. Conceptually, a skip list represents the set of elements as a number of ordered chains equal to its height. All elements appear in the bottom chain, and each level above that contains a fraction p of the elements in the chain below it. Operations defined on this structure, illustrated in Figure 1, have expected logarithmic time.

A perfectly balanced skip list of order k requires that every kth node of height at least h is of height at least h+1 [9]. In the probabilistic case, k=1/p. Two elements in a given chain are said to "skip" k-1 elements in the chain below them. The skipped elements form a gap. In order to give the skip list deterministic logarithmic time bounds, the perfect balance condition is relaxed to allow a variable width gap. A 1-2 skip list, for example, allows a gap of 1 or 2 and has a corresponding 2-3 tree [7].

Searching in a skip list begins at the head of the highest level chain, and at each point the decision is

made to move either down or right. At each level, the search proceeds right as long as the next key at the current level is less than the search key, then it moves down and repeats the process. There are never more than the gap size number of rightward motions before the search moves down.

The correspondence between search trees and skip lists is easy to generalize. The elements forming a gap of size k-1 correspond to a multi-way tree node with k-1 keys and k children. Top-down algorithms for insertion and deletion in 2-3-4 trees exist [5], and they are used as a basis for the top-down 1-2-3 skip list [9].

2.1 Top-down 1-2-3 Skip List

In its simplest form, the top-down 1-2-3 skip list uses a linked representation with fixed-size nodes, as illustrated in Figure 2. Several modifications have been made to the array representation. First, instead of using the next element's key for comparison, each key is stored at the node where it is actually used. Second, a new key named max that is greater than any key in the set is used to terminate each chain. The last key of any gap is always greater than the search key, and is called the high key. Though a comparison against the high key is redundant computation, it simplifies the code and has other uses as well. Additionally, there is a dummy head node and two sentinel nodes bottom and tail. Having made these modifications the representation is now somewhere between a skip list and a binary tree.

C code for search and lookup are given by Munro et al. using the following type definition:

```
typedef struct _Node Node;
struct _Node{
   int k;
   Node *r, *d;
};
```

Search is written as:

```
Node* search(int v){
   Node* x = head;
   bottom->k = v;
   while(v != x->k)
        x = (v < x->k)?x->d:x->r;
   return x;
}
```

The top-down insertion routine uses a precondition before descending into a gap ensuring that it can perform any necessary splits at the next level down. A full gap (of size three, in this case) must be split (into two gaps of size one) in order to meet this condition. In order to test whether the node x's gap is full, it uses the expression (x->k == x->d->r->r->r->k). The function returns 1 on a successful insertion and 0 on failure:

```
int insert(int v){
  node *t, *x = head;
  bottom->key = v;
  for(; x \neq bottom; x = x - d)
    while(v > x - > k)
       x = x - r;
    if(x \rightarrow d == bottom \&\&
        v == x - key
       return 0;
    if(x->d == bottom ||
        x - k = x - d - r - r - k
       t = new(Node);
       t - r = x - r;
       t \rightarrow d = x \rightarrow d \rightarrow r \rightarrow r;
       x - r = t;
       t - k = x - k;
      x - k = x - d - r - k;
     }
  }
  if(head->r != tail){
    t = new(Node);
    t \rightarrow d = head;
    t \rightarrow r = tail;
    t \rightarrow k = max;
    head = t_i
  }
  return 1;
}
```

Deletion uses the opposite precondition; it must ensure that it can perform any necessary concatenations at the next level down. A gap at minimum capacity (which is tested with the expression (x->k == x - >d - >r - >k) is dealt with in four cases. An adjacent gap must be found either to the right or the left, these two cases are treated differently. If the adjacent gap is also at minimum capacity then they are concatenated to form a single gap of size three. If the adjacent gap is not at minimum capacity then an element is borrowed from that gap. When a match is found at the bottom of the skip list the node is removed from the list in one of two ways, depending on whether it is the first element of a gap or not. Finally, the first interior node with a key that matches the deleted key must be revisited and a second downward pass replaces the deleted key with the next greatest value in the set. Code for delete is supplied in Appendix A.

Note that there is no recursion, nor the use of a stack or parent pointers in this code, therefore it is easier to understand. Bottom-up algorithms, on the other hand, require one of these mechanisms to keep track of nodes that have been visited, and this generally complicates program structure.

2.2 Higher order top-down skip lists

Higher order skip lists are constructed by allowing larger gaps. In general a skip list that allows a gap of (m/2)-1, ..., (m-1) corresponds to a B-tree of order m. To obtain the storage efficiency of a B-tree, however, it is necessary to place all the keys and down pointers in a gap into the same "page". Since all the keys in a skip list are stored in the bottom chain, the corresponding B-tree is actually a B⁺-tree. Using this representation, the paged 1-2-3 skip list would appear as in Figure 3. The only structural differences between the paged skip list and the B⁺-tree are:

- The high key occupies one extra key per skip list node
- Right links are used on internal and external nodes, whereas the B⁺-tree has them only on external nodes.

Other than these, the only thing that distinguishes the two data structures is that B-trees are bottom-up while the skip list is top-down.

Rewriting the top-down algorithms given above to use the paged representation is surprisingly simple. Deletion is simplified by using an explicit count instead of key comparisons to detect overflow and underflow, making it is no longer necessary to revisit internal nodes after a key is deleted. Deleted keys are allowed to remain in the skip list just as in the B⁺-tree. The borrowing operations are replaced with redistribution operations, to reduce the chances of future underflow.

The presence of right links in the paged skip list is a matter of convenience only, in order to support sequential access. Internal nodes have the same format as external nodes, so their right links are maintained as well, but they are not actually used by any of the dictionary operations.

2.3 Other concerns and applications

Concurrency is very important in database applications. Top-down algorithms are inherently concurrent, since they only require locking a fixed-size context around the current node during any operation. A related issue when secondary storage is involved is the efficient handling of blocked writers (allowing readers to pass through). It is interesting to note that the two structural extensions used in Lehman and Yao's B^{link} -tree for efficient concurrency in B^* -trees are the same two used in the skip list–a high key and right links on internal nodes [8]. Using their scheme, no read locks are required because the high key and right pointer allow a reader to find the adjacent sibling in case a concurrent modification has occurred. Their techniques should apply to top-down algorithms in a similar fashion.

The skip list thus presented can serve as a priority queue without modification since the minimum key can be easily found in constant time and deleted in logarithmic time [2].

3. Experimental Results

The linked skip list, paged skip list, B+-tree, AVL tree, and treap were each programmed in C. Two variations on the B⁺-tree were tested, one using binary search within the node and one using sequential search. The paged skip list and B⁺-tree allow their node size to be varied at compile-time in order to study the effect of tuning the data structure to the cache-block size. For plotting results as a function of node size, the non-variable-size structures are reported for every node size tested and appear as a flat line. Only node sizes that are a power of two were tested. For simplicity, it was assumed that each key, datum, and pointer occupies one 32-bit word. A cache-block of size 2m words can contain a B+-tree of order m. Due to the extra space occupied by the high key in the skip list, its branching factor is always one less than the corresponding B⁺tree. For example, a 32-word cache block has m=16and a node size of 128 bytes.

An experiment was constructed using the UC Berkeley Home IP HTTP traces [4]. These traces consist of a total of 9,244,728 client HTTP requests taken over a period of 18 days. A keys was generated from each request as follows. The high 16-bits of each key were taken as a hash of the server's IP address, and the low 16-bits were taken as a hash of the request string. There are 2,663,855 unique keys using this method. A tree is built by inserting every unique key into the tree in random order, then searching for each key in the order it appears in the trace, and then deleting each key in random order. The intent of this setup was to preserve any spatial and temporal locality in the trace for the search portion of the workload. Keys are clustered spatially by the server each request was bound for, and have whatever temporal locality that was present in the original traces.

The experiments were run on an Intel Pentium II Xeon processor running at 450 MHz and simulated on the SimpleScalar architecture. Since the SimpleScalar out-of-order simulator is quite slow, a short 4-hour fragment of the traces were used for simulation. The shortened data set contains 95,769 requests and 61,308 unique keys.

Figure 4 shows the results of running the experiment on three processor configurations: the Intel processor and the SimpleScalar processor with memory latencies of 18 (the default value) and 100 cycles. The SimpleScalar processor was configured with a 32K split L1 I- and D-cache with 64 byte blocks and a 256K unified L2 cache with 128 byte blocks. The AVL tree is the best performing fixednode-size data structure, and the paged skip list is the best performing variable-node-size data structure. Moreover, the gap between the cache-conscious data structures and their counterparts is growing, as shown by the 100 cycle memory latency configuration. The paged skip list outperforms the sequential-search B⁺-tree by 12% on the Intel processor.

Table 1 lists various quantities describing each data structure for a node size of 128 bytes on the SimpleScalar processor. These values depend only on the specific algorithms used and not the memory latency. The cache-conscious data structures not only improve access time, but they also improve storage efficiency. The storage ratio is given as the number of words of overhead per key/datum pair in the index. The two skip list implementations require 31% fewer instructions, on average, than the bottom-up methods.

Table 2 lists the cache performance of each data structure. The three cache-conscious data structures produce significantly fewer cache misses in both levels of the cache, by a factor of five or more in the L2 cache. These values vary only slightly with memory latency, since latencies can effect the out-of-order pipeline. They were measured for the 18 cycle memory latency configuration.

Table 3 shows the execution time for each data structure with memory latencies of 18, 100, and 500 cycles and the corresponding cycles per instruction (CPI). The cache-conscious data structures are

result in a 48% improvement in total cycles, on average, compared to their counterparts, with a memory latency of 18 cycles. Increasing the memory latency to 100 cycles (500 cycles) leads to an improvement of 77% (84%). These results indicate that the performance of the cache-conscious data structures is steadily increasing relative to the others.

Figure 5 shows the results of carrying the experiment out for large node sizes on the full data set with the binary-search B⁺-tree, the paged skip list, and the AVL tree. Large nodes are inefficient for two reasons. First, their cache behavior is identical to a binary search tree, so the binary B⁺-tree approaches the AVL tree for searching with large node sizes. Second, the cost of update operations becomes dominated by the cost of memory-to-memory copies for updating individual nodes. These copies also disrupt the cache. Node sizes larger than 1024 bytes (m=128) have worse performance on insert and delete than the AVL tree due to this effect. The cross-over point at which binary search begins to outperform sequential search occurs at a node size of 2048 bytes (*m*=256).

4. Conclusion

Cache performance is a growing bottleneck for dictionary data structures. The cache-conscious data structures measured in this study already outperform their counterparts by 48%. This value will increase steadily over the next few years to 77% or more when cache misses begin to cost hundreds of processor clock cycles.

Data structures that are optimized for reducing the number of accesses to secondary storage will be effected by this trend as well since they have the poor cache-performance of a binary search. There is currently no work that we know of that adopts a two-level strategy for reducing both cache and disk accesses. Optimizing for both levels of the storage hierarchy will increase in importance as the cost of a cache miss increases.

The top-down algorithms for maintaining skip lists are easier to understand and implement than the corresponding bottom-up algorithms for B-trees, and they reduce the instruction count as well. Given that their performance is similar in main memory, the paged skip list seems like a good alternative to Btrees.

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Appendix A: Code for delete

```
int delete(int k){
  Node *1, *t, *t2, *r, *x;
  Node *f = NULL;
  int d = 0;
```

```
for(l = NULL, x = head->d;
    x != bottom;
    1 = NULL, x = x -> d)
  for (; k > x - > k;
         l = x, x = x - r { }
  if(x \rightarrow d == bottom)
    if(k == x -> k){
       if(l == NULL){
         r = x - r;
         x - k = r - k;
         x - r = r - r;
         free(r);
       } else {
         1->r = x->r;
         free(x);
         x = l;
       }
       if(f)
         fixup(f,k,x->k);
       d = 1;
    }
  } else {
    if(k == x -> k \&\& f == NULL)
       f = x;
    if(x->d->r->k == x->k)
       if(l == NULL){
         r = x - r;
         if(r - > d - > r - > k == r - > k)
           x \rightarrow k = r \rightarrow k;
           x \rightarrow r = r \rightarrow r;
            free(r);
         } else {
           x - k = r - d - k;
           r \rightarrow d = r \rightarrow d \rightarrow r;
         }
       } else {
         if (1 - > d - > r - > k == 1 - > k)
           1 - k = x - k;
           1->r = x->r;
            free(x);
         } else {
            t = 1 - > d;
           while(t->r != x->d){
              t2 = t;
              t = t - r;
            }
            1 - k = t2 - k;
           x \rightarrow d = t;
         }
       }
    }
  }
}
```

if(head->d->r == tail){

```
t = head;
                                               x = bottom; x = x->d
   head = head->d;
                                             while(oldk > x->k)
    free(t);
                                               x = x->r;
  }
                                             if(x \rightarrow k == oldk)
                                               x->k = newk;
 return d;
}
                                           }
void fixup(Node *f,
                                         }
           int oldk, int newk){
 Node *x;
  for(x = head ->d;
```



A skip list is comprised of a hierarchy of chains. Every key in the set resides in the bottom chain, and each level above that contains a fraction p of the elements in the chain below it.





The linked skip list representation moves keys into the node where they are actually used and adds a high key.

Figure 2: A skip list with the linked list representation



The skip list (a) resembles a B^+ -tree (b) except that it has a high key and right links at all nodes, not just in the external nodes.





Figure 4: Performance comparison for three processor configurations



These graphs show the time taken by the dictionary operations for the full data set on the Intel processor. Searching in large nodes has the same performance as the (binary) AVL tree. For large nodes the cost of insertion and deleteion becomes dominated by the cost of memory-to-memory copies when updating individual nodes (which also disrupts the cache). They show the node size at which binary-search begins to outperform sequential search. This indicates that choosing disk-sized pages without regard for cache-performance incurs a serious (and growing) performance penalty in main memory.

Figure	5:	Large	node	sizes
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Tree	Height	Storage Ratio	Inst. Count	Loads	Stores	Branches
AVL	19	2.50	141M	38.3M (27%)	21.8M (15%)	34.3M (24%)
Treap	0	2.50	124M	34.4M (28%)	19.2M (15%)	29.1M (23%)
Skip List (P)	4	1.69	88.3M	19.6M (22%)	6.63M (7.5%)	17.7M (20%)
Skip List (L)	13	3.14	87.9M	30.9M (35%)	5.91M (6.7%)	23.1M (26%)
B+Tree (S)	4	1.66	125M	28.1M (22%)	14.2M (11%)	26.0M (21%)
B+Tree (B)	4	1.66	123M	26.7M (22%)	14.2M (12%)	25.5M (21%)

Table 1: Various quantities for each data structure (node size = 128 bytes)

Tree		L1		L2			
	Accesses	Misses	Writeback	Accesses	Misses	Writeback	
AVL	62.5M	2.04M (3.3%)	1.12M (1.8%)	3.17M	910K (29%)	521K (16%)	
Treap	56.8M	3.30M (5.8%)	1.47M (2.6%)	4.77M	1.48M (31%)	706K (15%)	
Skip List (P)	27.5M	880K (3.2%)	364K (1.3%)	1.99M	220K (11%)	153K (7.7%)	
Skip List (L)	42.7M	4.10M (9.6%)	507K (1.2%)	4.61M	2.02M (44%)	367K (8.0%)	
B+Tree (S)	43.9M	888K (2.0%)	385K (0.88%)	1.96M	223K (11%)	156K (8.0%)	
B+Tree (B)	41.1M	882K (2.1%)	383K (0.93%)	1.51M	221K (15%)	156K (10%)	

 Table 2: Cache performance for each data structure (node size = 128 bytes)

Tree	Inst. Count	18 cycles		100 cycles		500 cycles	
		Cycles	СРІ	Cycles	СРІ	Cycles	СРІ
AVL	141M	126M	0.89	305M (+142%)	2.16 (+142%)	767M (+508%)	5.42 (+508%)
Treap	124M	155M	1.25	556M (+258%)	4.48 (+258%)	1.48G (+851%)	11.9 (+851%)
Skip List (P)	88.3M	64.0M	0.72	102M (+59%)	1.15 (+59%)	201M (+215%)	2.28 (+215%)
Skip List (L)	87.9M	162M	1.85	650M (+300%)	7.40 (+300%)	1.82G (+1021%)	20.7 (+1021%)
B+Tree (S)	125M	78.0M	0.62	115M (+48%)	0.92 (+48%)	217M (+179%)	1.74 (+179%)
B+Tree (B)	123M	89.8M	0.73	127M (+41%)	1.03 (+41%)	229M (+155%)	1.87 (+156%)