1. Professor Curly conjectures that if we do union by rank, without path compression, the amortized cost of all operations is $o(\log n)$. Prove him wrong by showing that if we do union by rank without path compression, there can be $m$ MAKESET, UNION and FINDSET operations, $n$ of which are MAKESET operations, where the total cost of all operations is $\Theta(m \log n)$.

2. Problem 17-2 (Making Binary Search Dynamic)

3. Problem 22-4 (Reachability)$^1$

4. Professor Moe conjectures that for any connected graph $G$, the set of edges $T=\{(u,v) : \text{there exists a cut } (S,V-S) \text{ such that } (u,v) \text{ is a light edge crossing } (S,V-S)\}$ always forms a minimum spanning tree. Given a simple example of a connected graph that proves him wrong.

5. Consider a connected graph $G = (V, E)$ with positive edge weights. Call a subset of edges, $F$, a cycle cover if every cycle in $G$ contains at least one edge in $F$. In other words, removing the edges of $F$ from $G$ results in an acyclic graph. You want to find a cycle cover, $F$, of $G$ with minimum weight, i.e. the sum of the weight of all edges in $F$ is minimized over all cycle covers. Give an efficient algorithm to solve this and give the runtime of your algorithm as a function of $n = |V|$ and $m = |E|$. Hint: Think about the maximum-weight spanning tree problem.

6. Professor Matsumoto conjectures the following converse of the safe-edge theorem:
   Let $G = (V, E)$ be a connected, undirected weighted graph, with

---

$^1$The answer to this problem can be used in an efficient randomized algorithm for estimating the *number* of vertices that are reachable.
weight function \( w \). Let \( A \) be a subset of \( E \) that is included in some minimum spanning tree for \( G \). Let \( (S, V - S) \) be any cut of \( G \) that respects \( A \) and let \((u, v)\) be a safe edge for \( A \) that crosses \((S, V - S)\). Then \((u, v)\) is a light edge for the cut.

Is this conjecture true? If so, prove it. If not, give a counterexample.

7. Prove that if an edge \((u, v)\) is in a MST for a graph \( G \), then \((u, v)\) is a light edge crossing some cut in \( G \).